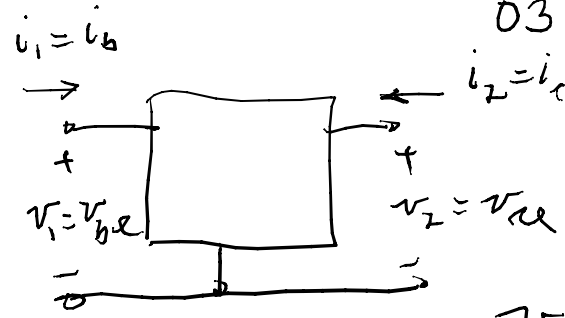
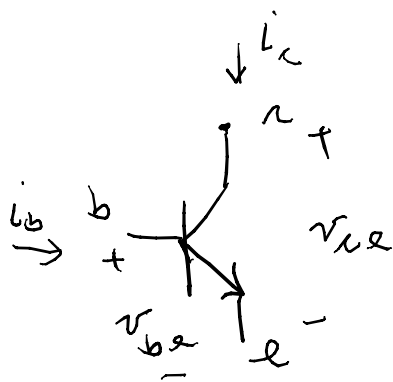
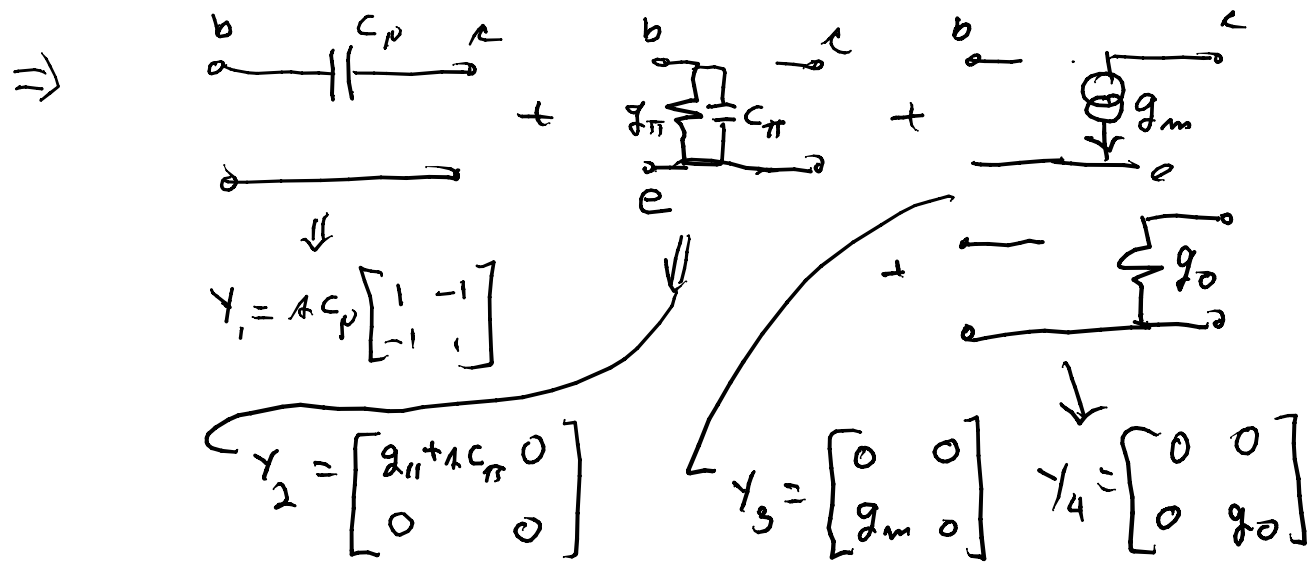
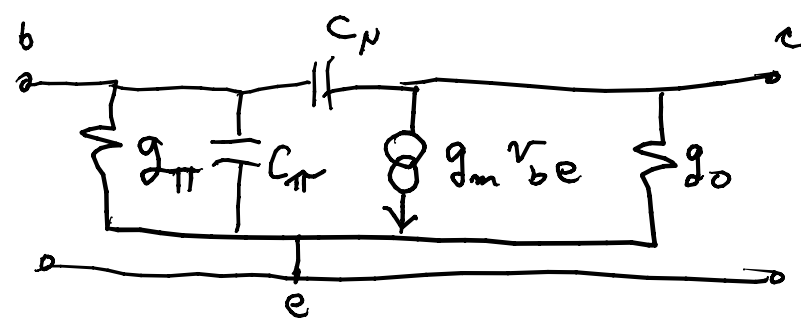


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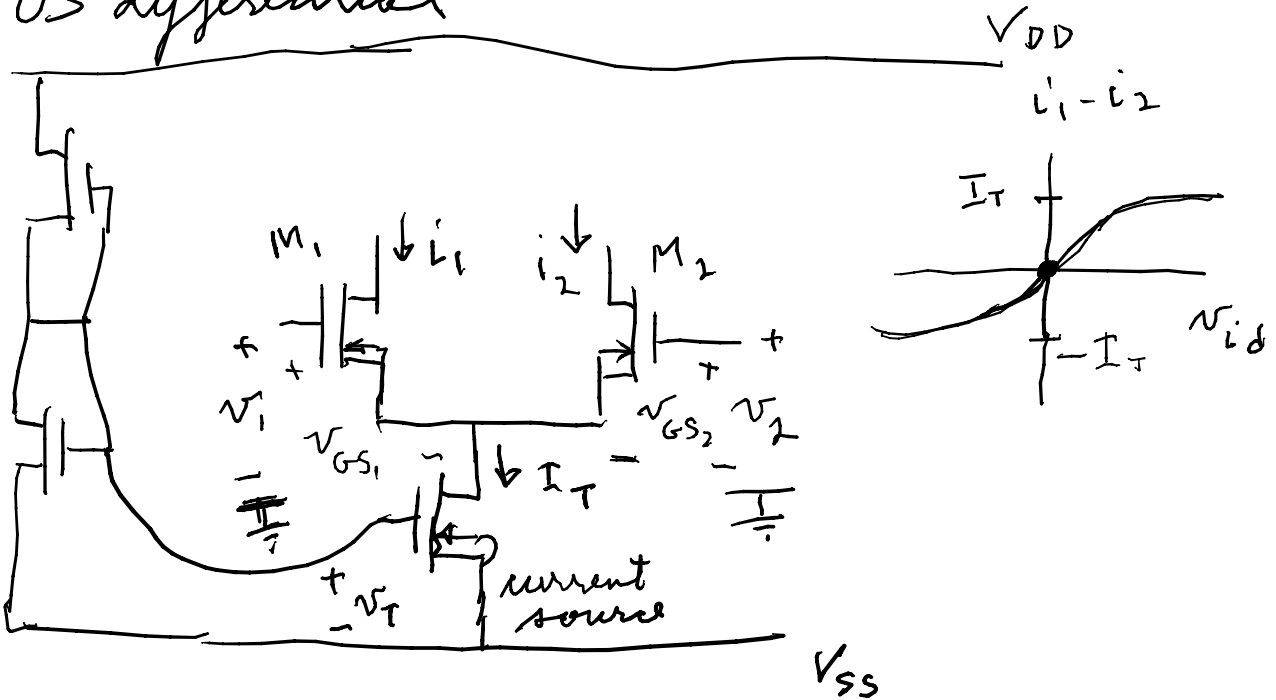


$$i = \begin{bmatrix} i_b \\ i_c \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Y \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_{be} \\ v_{ce} \end{bmatrix}$$



$$Y \approx \begin{bmatrix} g_\pi + A(C_\pi + C_\mu) & -AC_\mu \\ g_m - AC_\mu & g_o + AC_\mu \end{bmatrix} \quad \text{for the BJT and emitter}$$

MOS differential



$$v_{GS1} - v_{GS2} = v_{id} \quad ; \text{ assume } I_T \text{ fixed}$$

$$I_T = i_1 + i_2$$

assume V_{DD} & V_{SS} are large enough to keep M_1 & M_2 in saturation (common mode voltage small)

$$i_1 = \frac{KP}{2} \frac{W}{L} (v_{GS1} - V_{TO})^2 = \beta (v_{GS1} - V_{TO})^2 \quad \beta = \frac{KPW}{2L}$$

$$i_2 = \frac{KP}{2} \frac{W}{L} (v_{GS2} - V_{TO})^2 = \beta (v_{GS2} - V_{TO})^2$$

$$1) \quad v_{id} = \sqrt{\frac{i_1}{\beta}} - \sqrt{\frac{i_2}{\beta}} \quad ; \quad I_T = i_1 + i_2 \Rightarrow i_2 = I_T - i_1 \quad 2)$$

$$2) \rightarrow 1) = 3) \quad v_{id} = \sqrt{\frac{i_1}{\beta}} - \sqrt{\frac{I_T - i_1}{\beta}} \Rightarrow 3) = 4) \quad v_{id}^2 = \frac{i_1}{\beta} + \left(\frac{I_T - i_1}{\beta} \right) - 2 \sqrt{\frac{i_1(I_T - i_1)}{\beta^2}}$$

corrected: squaring

$$4 \left(\frac{I_1 (I_T - I_1)}{\beta^2} \right) = \left(v_{id}^2 - \frac{I_T}{\beta} \right)^2 = \frac{I_T^2}{\beta^2} - 2 \frac{I_T}{\beta} v_{id}^2 + v_{id}^4$$

$$\text{or } \frac{4}{\beta^2} I_1^2 - \left(\frac{4 I_T}{\beta^2} \right) I_1 + \left(v_{id}^2 - \frac{I_T}{\beta} \right)^2 = 0$$

$$\text{or } I_1^2 - I_T \cdot I_1 + \left(\frac{2}{\beta} v_{id}^2 - \frac{I_T}{2} \right)^2 = 0$$

$$\text{or } I_1 = \frac{I_T}{2} \pm \frac{1}{2} \sqrt{I_T^2 - 4 \left(\frac{I_T^2}{4} - \frac{2}{\beta} I_T v_{id}^2 + \frac{4}{\beta^2} v_{id}^4 \right)}$$

$$\text{or } I_1 = \frac{I_T}{2} + v_{id} \sqrt{\frac{2 I_T}{\beta} - \left(\frac{2 v_{id}}{\beta} \right)^2}$$

$$I_2 = \frac{I_T}{2} - v_{id} \sqrt{\frac{2 I_T}{\beta} - \left(\frac{2 v_{id}}{\beta} \right)^2}$$

where + is
chosen on $\sqrt{\quad}$
as $I_1 \uparrow$ as $v_{id} \uparrow$
the other root
gives I_2

$$\therefore I_0 = I_1 - I_2 = 2 v_{id} \sqrt{\frac{2 I_T}{\beta} - \left(\frac{2 v_{id}}{\beta} \right)^2} \quad \text{but term under } \sqrt{\quad} \text{ must not be } < 0$$

as $I_1 - I_2$ does not decrease with v_{id} we seek
the max of I_0 w.r. v_{id} after which $I_0 = I_T$

$$\text{so the max is for } I_T = 2 V_m \sqrt{\frac{2 I_T}{\beta} - \left(\frac{2 V_m}{\beta} \right)^2}$$

$$\left(\frac{\beta I_T}{2 V_m^2} \right) = 2 \sqrt{\left(\frac{\beta I_T}{2 V_m^2} \right) - 1} \Rightarrow x = 2 \sqrt{x-1} \Rightarrow x^2 - 4x + 4 = 0 \quad (x-2)^2$$

$$\text{or } \frac{\beta I_T}{2 V_m^2} = 2 \Rightarrow V_m = \frac{1}{2} \sqrt{\beta I_T}$$

$$\therefore i_{out} = \begin{cases} I_T & \text{for } \frac{1}{2}\sqrt{\beta I_T} \leq v_{id} \\ 2v_{id}\sqrt{2\frac{I_T}{\beta} - \left(\frac{2v_{id}}{\beta}\right)^2} & \text{for } -\frac{1}{2}\sqrt{\beta I_T} \leq v_{id} \leq \frac{1}{2}\sqrt{\beta I_T} \\ -I_T & \text{for } v_{id} \leq -\frac{1}{2}\sqrt{\beta I_T} \end{cases}$$

