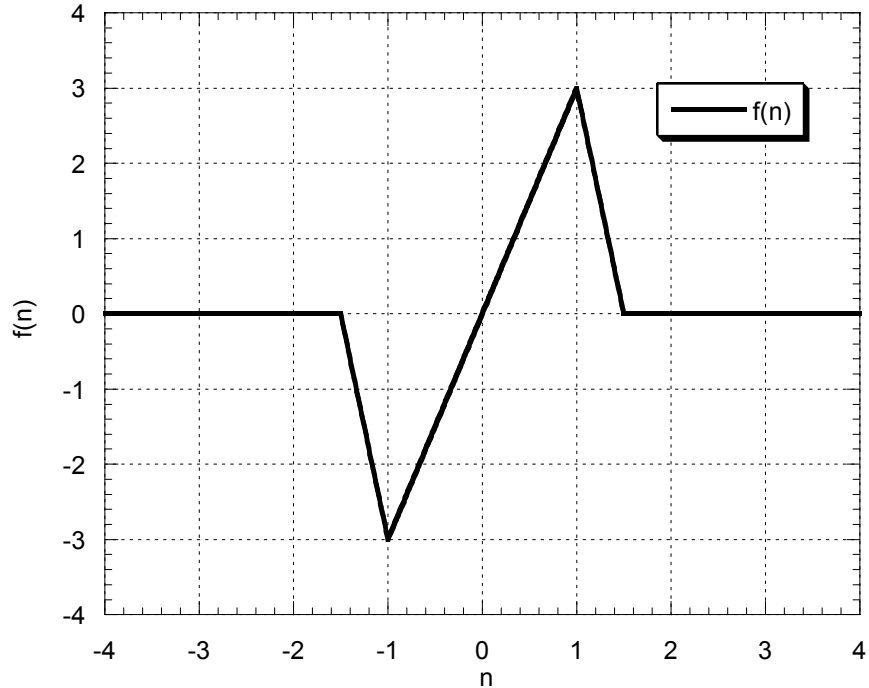


Problem 1: (30)

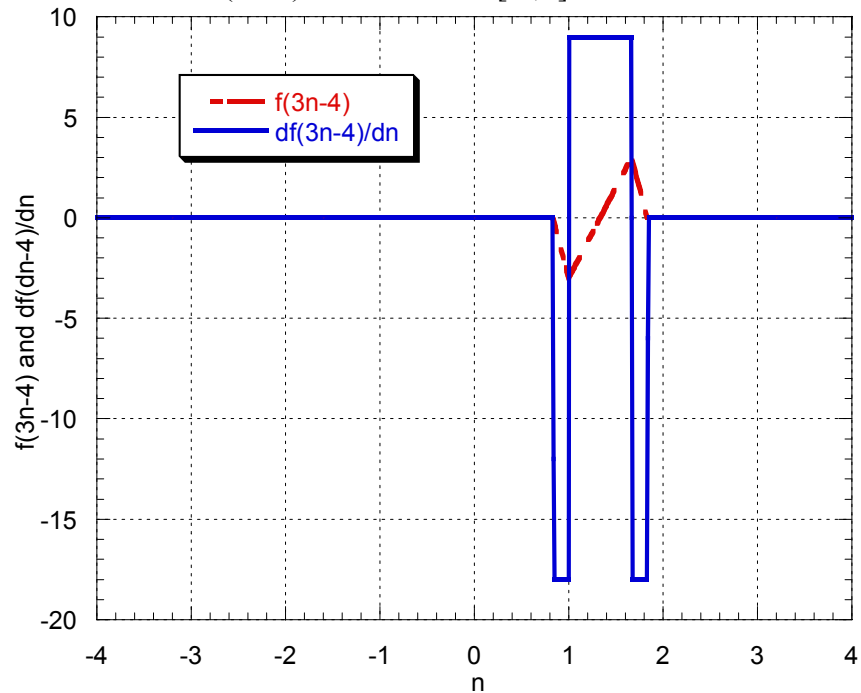
$$f(n) = 3[-|n+1.5| + 1.5|n+1| - 1.5|n-1| + |n-1.5|]$$

a) sketch $f(n)$ in the interval $[-4, 4]$:



Important transition points: $(-1.5, 0)$, $(-1, -3)$, $(1, 3)$, and $(1.5, 0)$.

b) sketch the derivative of $f(3n-4)$ in the interval $[-4, 4]$:



Important transition points:

$(5/6, 0 \text{ or } -18)$, $(1, -18 \text{ or } 9)$, $(5/3, 9 \text{ or } -18)$ and $(11/6, -18 \text{ or } 0)$.

c) Write $f(3n-4)$ as a weighted sum of shifted satlins functions:

Since $\text{satlin} = \frac{1}{2} \cdot [|n+1| - |n-1|]$,

$$f(n) = 3[-|n+1.5| + 1.5|n+1| - 1.5|n-1| + |n-1.5|]$$

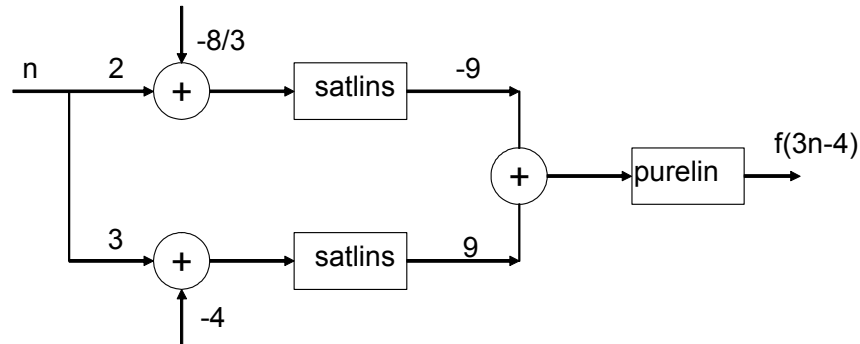
$$= -9 \cdot \left[\frac{1}{2} \cdot \left(\left| \frac{2}{3}n + 1 \right| - \left| \frac{2}{3}n - 1 \right| \right) \right] + 9 \cdot \left[\frac{1}{2} \cdot (|n+1| - |n-1|) \right]$$

Therefore:

$$f(n) = -9 \cdot \left[\text{satlin}\left(\frac{2}{3}n\right) - \text{satlin}(n) \right], \text{ and}$$

$$f(3n-4) = -9 \cdot \left[\text{satlin}\left(2n - \frac{8}{3}\right) - \text{satlin}(3n-4) \right]$$

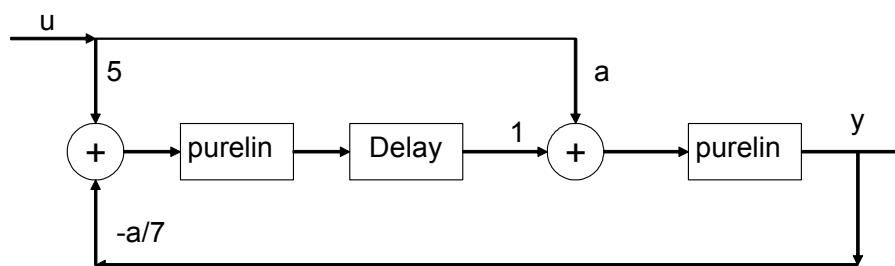
d) Draw a neural network which realizes $f(3n-4)$:



Problem 2: (35)

A digital filter: $\frac{y(z)}{u(z)} = \frac{a + 5Z^{-1}}{1 + (1/7)az^{-1}}$

a) Neural network realization:



b) Unit step function response:

$$y(t) = au(t) + 5u(t-1) - (a/7)y(t-1)$$

$$y(0) = 0$$

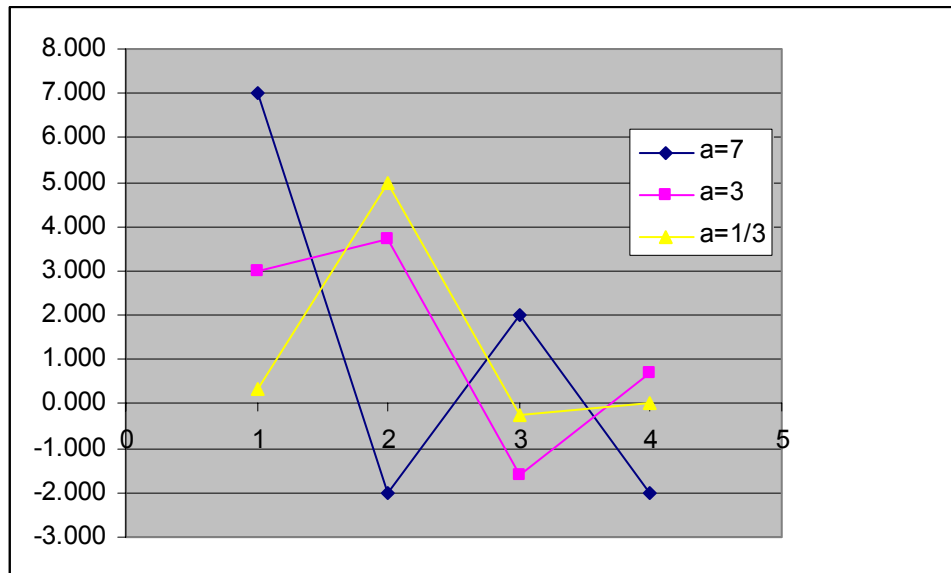
$$y(1) = a + 5 \cdot 0 - (a/7) \cdot 0 = a$$

$$y(2) = 0 + 5 \cdot 1 - (a/7) \cdot a = 5 - a^2/7$$

$$y(3) = 0 + 0 - (a/7) \cdot (5 - a^2/7) = -(a/7) \cdot (5 - a^2/7)$$

$$y(4) = 0 + 0 - (a/7) \cdot [-(a/7) \cdot (5 - a^2/7)] = (-a/7)^2 \cdot (5 - a^2/7)$$

t	u(t)	y(t, a)	y(t, a=7	y(t, a=3	y(t, a=1/3
0	0	0	0	0	0
1	1	a	7	3	1/3
2	0	$5-a^2/7$	-2	26/7	314/63
3	0	$(-a/7)(5-a^2/7)$	2	-78/49	-314/1323
4	0	$(-a/7)^2(5-a^2/7)$	-2	234/343	314/27783



c) (RWN) By using the time input to be n, kept in increments under $n=0.1$, and the training output to be $f(3n-4)$, the network of a can be trained by adjusting a. Since there is only one parameter, a, to adjust, the error can only be set to 0 at one value of n.

Problem 3: (30)

a) Calculate equilibrium points in terms of generic, W, b, ε.

$$\varepsilon \frac{dn(t)}{dt} = -n(t) + W \cdot a(t) + b \quad (\text{Equation 18.6 from text book, pp. 18-4})$$

$$a(t) = f(n(t)) \quad (\text{Equation 18.7 from text book, pp. 18-4})$$

At equilibrium point, $\frac{dn(t)}{dt} = 0$. Therefore $-n(t) + W \cdot a(t) + b = 0$.

with $a(t) = f(n(t)) = \text{purelin}(5n(t)) = 5n(t)$,

$$-n(t) + W \cdot a(t) + b = -n(t) - W \cdot 5 \cdot n(t) + b = 0,$$

at equilibrium, we have:

$$\begin{cases} n(t) = (I - 5W)^{-1} \cdot b \\ a(t) = 5 \cdot (I - 5W)^{-1} \cdot b \end{cases}$$

b) (RWN) This no longer needs to be a valid Lyapunov function since it needs not be bounded below since $f(\cdot)$ does not saturate, even though $dV(a)/dt < 0$ will hold when $W=W^T$.

$$c) \frac{1}{3} \cdot \frac{dn}{dt} = -5n + \begin{bmatrix} 2 & -3p \\ p & -1 \end{bmatrix} a + \begin{bmatrix} 5 \\ -9 \end{bmatrix}, \quad n = \begin{bmatrix} n1 \\ n2 \end{bmatrix}, \quad a = \begin{bmatrix} 5n1 \\ 5n2 \end{bmatrix}$$

At equilibrium, $dn/dt=0$,

$$-5 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} n1 \\ n2 \end{bmatrix} + \begin{bmatrix} 2 & -3p \\ p & -1 \end{bmatrix} \cdot \begin{bmatrix} 5n1 \\ 5n2 \end{bmatrix} + \begin{bmatrix} 5 \\ -9 \end{bmatrix} = 0,$$

$$-\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} n1 \\ n2 \end{bmatrix} + \begin{bmatrix} 2 & -3p \\ p & -1 \end{bmatrix} \cdot \begin{bmatrix} n1 \\ n2 \end{bmatrix} + \begin{bmatrix} 1 \\ -9/5 \end{bmatrix} = 0,$$

$$\begin{bmatrix} n1 \\ n2 \end{bmatrix} = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3p \\ p & -1 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 1 \\ -9/5 \end{bmatrix}$$

$$= \left(\begin{bmatrix} -1 & 3p \\ -p & 2 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 1 \\ -9/5 \end{bmatrix} = \frac{1}{3p^2 - 2} \cdot \begin{bmatrix} 2 & -3p \\ p & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -9/5 \end{bmatrix} = \begin{bmatrix} \frac{5.4p + 2}{3p^2 - 2} \\ \frac{p + 1.8}{3p^2 - 2} \end{bmatrix}$$

Therefore, equilibrium point: $n = \begin{bmatrix} \frac{5.4p + 2}{3p^2 - 2} \\ \frac{p + 1.8}{3p^2 - 2} \end{bmatrix}, \quad a = 5n.$

c) To look at the stability we look at zeros of the characteristic polynomial formed from $s \cdot \varepsilon \cdot n = -5 \cdot n + W \cdot 5 \cdot n + b \Rightarrow (s \cdot \varepsilon \cdot I_2 + 5 \cdot I_2 - 5 \cdot W) \cdot n = b$

$$P(s) = \det(s \cdot \varepsilon \cdot I_2 + 5 \cdot I_2 - 5 \cdot W) = \det \begin{bmatrix} s \cdot \varepsilon - 5 & 15p \\ -5p & s \cdot \varepsilon + 10 \end{bmatrix}$$

$$= (s \cdot \varepsilon)^2 + 5 \cdot (s \cdot \varepsilon) + (75p^2 - 50)$$

Roots: (always assume $\varepsilon > 0$.)

$$(s \cdot \varepsilon)_{1,2} = -\frac{5}{2} \pm \frac{5}{2} \cdot \sqrt{9 - 12p^2}.$$

i) For $|p| > \sqrt{3/4}$, $(s \cdot \varepsilon)_{1,2} = -\frac{5}{2} \pm \frac{5}{2} \cdot j \cdot \sqrt{12p^2 - 9}$, $\text{Re}(s_{1,2}) < 0$, stable.

ii) For $\sqrt{2/3} \leq |p| \leq \sqrt{3/4}$, $(s \cdot \varepsilon)_{1,2} \leq 0$, stable.

iii) For $|p| < \sqrt{2/3}$, one of $(s \cdot \varepsilon)_{1,2} > 0$, not stable.

There form when $|p| \geq \sqrt{2/3}$, stable.