

ENEE434 Spring 2005 Final Exam

100 points; 120 minutes, open book, open notes; if stuck go on to the next; good luck!
Show your work for partial credit.

1. [50 points, 45 minutes]

A new activation function is proposed for all real n to be

$$a(n) = \frac{n}{1 + |n|}$$

- Sketch $a(n)$ and its derivative versus n labeling important points.
- If possible find the inverse and if not possible give the reason.
- By use of weights this normalized form can be transformed to

$$A(x) = \frac{kx}{1 + d|x|}$$

Draw a neural network with x as input, $a(n)$ as activation function, and output of $y=A(x)$.

- For the neural network of part c) determine k and d to minimize the rms error for approximating $y=x^3$ for $x=0, \pm 1, \pm 2$ by $A(x)$.
- For backpropagation when signals are complex valued, $a(n)$ is useful when extended to complex n . However it is not an analytic function of the complex variable n . Show that there is a complex n for which no derivative exists and discuss the meaning of this fact.

2. [50 points, 45 minutes]

A neural network takes three complex numbers, $P_1=0+j0$, $P_2=20+j0$, $P_3=10+j20$ ($j=\sqrt{-1}$) as inputs at $t=0$ after which the inputs remain zero. The network generates recursively three complex numbers, $z_1(t)$, $z_2(t)$, $z_3(t)$, [of the complex form $z=x+jy$] as outputs for all nonnegative integer times t . Assuming $z_i(0)=P_i$, $i=1,2,3$, the network is described by

$$z_i(t+1)=a \cdot z_i(t)+(1-a) \cdot z_{i+1}(t); i=1,2,3$$

where $0 < a < 1$ and the subscripts are taken mod(3) [that is $z_4(.)=z_1(.)$]

- Represent the neural network in state-variable matrix form

$$x(t+1)=Ax(t)+Bu(t), y(t)=Cx(t)$$

where $x=[z_1, z_2, z_3]^T$, $u=\text{input}$, $y=\text{output}=x$.

- Draw the neural network using delays, weights, and activation functions.
- At each instant of time the three $z_i(t) = x_i(t)+jy_i(t)$ can be considered as representing a triangle.

On the attached paper with grids label the vertices and draw these triangles for $t=0,1,2,3,4$ when $a=1/4$.

- Prove that for $0 < a < 1$ the triangles converge to a point and discuss the physical meaning of that point. What happens for other real a ?