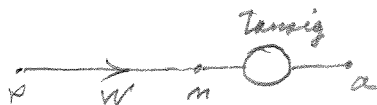


a)



$$a(u) = \text{tansig}(m(k))$$

$$m(k) = W(k) \cdot r$$

$$W(1) = W(0) - \alpha a(0) a(0)^T; \quad a(0) = a^M(0) = -2 \hat{F}^M(m^M(0)) (t - a(0))$$

$$\hat{F}^M(m) = \begin{bmatrix} \frac{d \text{tansig}(m_1)}{dm_1} & 0 \\ 0 & \frac{d \text{tansig}(m_2)}{dm_2} \end{bmatrix} = \begin{bmatrix} 1 - a_1^2 & 0 \\ 0 & 1 - a_2^2 \end{bmatrix}$$

$$a_i(0) = \text{tansig}(m_i(0)) = \tanh(m_i(0)); \quad m(0) = \begin{bmatrix} m_1(0) \\ m_2(0) \end{bmatrix} = W(0) r = I r = r$$

$$1 - a_i^2 = 1 - \left(\frac{e^{2m_i} - 1}{e^{2m_i} + 1} \right)^2 = \frac{(e^{2m_i} + 1)^2 - (e^{2m_i} - 1)^2}{(e^{2m_i} + 1)^2} = \frac{e^{4m_i} + 2e^{2m_i} + 1 - (e^{4m_i} - 2e^{2m_i} + 1)}{(e^{2m_i} + 1)^2} = \frac{4e^{2m_i}}{(e^{2m_i} + 1)^2}$$

$$\therefore 1 - a_i(0)^2 = \frac{4e^{2r_i}}{(e^{2r_i} + 1)^2}$$

Fourthes

$$(t_i - a_i(0)) = (t_i - \tanh r_i) \Rightarrow (t - a(0)) = \begin{bmatrix} t_1 - \tanh r_1 \\ t_2 - \tanh r_2 \end{bmatrix}$$

 $\Rightarrow$

$$a(0) = -2 \hat{F}^M(m(0)) \cdot (t - a(0)) = -2 \begin{bmatrix} \frac{2}{e^{2r_1} + 1} & 0 \\ 0 & \frac{2}{e^{2r_2} + 1} \end{bmatrix} \begin{bmatrix} t_1 - \tanh r_1 \\ t_2 - \tanh r_2 \end{bmatrix}$$

 $\therefore$

$$W(1) = W(0) - \alpha a(0) a(0)^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \alpha (-2) \begin{bmatrix} \frac{4e^{2r_1}}{(e^{2r_1} + 1)^2} (t_1 - \tanh r_1) \\ \frac{4e^{2r_2}}{(e^{2r_2} + 1)^2} (t_2 - \tanh r_2) \end{bmatrix} \begin{bmatrix} r_1 & r_2 \end{bmatrix}$$

 α

$$\textcircled{*} W(1) = \begin{bmatrix} 1 + \frac{8\alpha r_1 e^{2r_1}}{(e^{2r_1} + 1)^2} (t_1 - \tanh r_1) & \frac{8\alpha r_2 e^{2r_1}}{(e^{2r_1} + 1)^2} (t_1 - \tanh r_1) \\ \frac{8\alpha r_1 e^{2r_2}}{(e^{2r_2} + 1)^2} (t_2 - \tanh r_2) & 1 + \frac{8\alpha r_2 e^{2r_2}}{(e^{2r_2} + 1)^2} (t_2 - \tanh r_2) \end{bmatrix}$$

b) Numerically for $v = \begin{bmatrix} +1/4 \\ -3/8 \end{bmatrix}$, $t = \begin{bmatrix} -1/2 \\ +1/8 \end{bmatrix}$

then $\tanh v_1 = \tanh(1/4) \approx 0.2449$; $t_1 - \tanh v_1 = -0.7449$

$\tanh v_2 = \tanh(-3/8) \approx -0.3584$; $t_2 - \tanh v_2 = 0.4834$

$\frac{e^{2v_1}}{(e^{2v_1} + 1)^2} = 0.2350$

$\frac{e^{2v_2}}{(e^{2v_2} + 1)^2} = 0.2179$

$\therefore W(1)_{1,1} = 1 + 8\alpha(0.250)(0.2350)(-0.7449)$
 $= 1 - \alpha(0.3501)$

$W(1)_{1,2} = -\alpha(-0.375)(0.2350)(-0.7449)$
 $= \alpha(0.5251)$

$W(1)_{2,1} = 8\alpha(0.250)(0.2179)(0.4834)$
 $= \alpha(0.2107)$

$W(1)_{2,2} = 1 + 8\alpha(-0.375)(0.2179)(0.4834)$
 $= 1 - \alpha(0.3160)$

$\therefore$

*

$W(1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \alpha \begin{bmatrix} -0.3501 & 0.5251 \\ 0.2107 & -0.3160 \end{bmatrix}$