

a) We require $V_{GS_n} \geq V_{TO_n}$ for M_n to be turned on. Thus

$$V_{in} \geq \frac{1}{2} V = V_{TO_n}$$

b) For M_p in saturation, $|V_{DS}| \geq |V_{GS}| - |V_{TO_p}|$

$$|V_{DS}| = V_{dd} - V_{out} \geq (V_{dd} - 0) - 1 = V_{dd} - 1$$

$$\Rightarrow -V_{out} \geq -1 \text{ or } 1 \geq V_{out}$$

$\therefore$ we need to find V_{out} vs V_{in}

For M_p in saturation

$$I_{S_p} = \frac{K_P W}{2 L} (|V_{GS_p}| - |V_{TO_p}|)^2 = \frac{K_P W}{2 L} (V_{dd} - 1)^2$$

for which we need

$$= I_{D_n} = \frac{K_P W}{2 L} \begin{cases} (V_{in} - V_{TO_n})^2 & \text{if } M_n = \text{sat} \Rightarrow V_{out} \geq V_{in} - V_{TO_n} \\ 2(V_{in} - V_{TO_n})V_{out} - V_{out}^2 & \text{if } M_n = \text{ohmic} \Rightarrow V_{out} \leq V_{in} - V_{TO_n} \end{cases}$$

Case $M_n = \text{sat}$:

$$V_{dd} - |V_{TO_p}| = \pm (V_{in} - V_{TO_n}) \quad (\text{use + sign as } M_n \& M_p \text{ on so } V_{in} \geq V_{TO_n})$$

$$\Rightarrow V_{in} = V_{dd} + V_{TO_n} - |V_{TO_p}| = 5 + \frac{1}{2} - 1$$

$$\Rightarrow V_{in} = 4.5 \quad (\text{necessary for } M_n \& M_p \text{ in saturation})$$

but we also need $1 \geq V_{out}$

$$\text{while } V_{out} \geq V_{in} - V_{TO_n} = V_{in} - \frac{1}{2} = 4.5 - 0.5 = 4.0$$

as $V_{out} \geq 4.0$ contradicts $1 \geq V_{out}$.

then M_n sat when M_p sat does not occur.

Case M_n ohmic:

$$I_{S_p} = I_{D_n} \Rightarrow \frac{K_P W}{2 L} (V_{dd} - |V_{TO_p}|)^2 = \frac{K_P W}{2 L} (2(V_{in} - V_{TO_n})V_{out} - V_{out}^2)$$

From this we can solve for V_{out} vs V_{in} and see for what V_{in} is $V_{out} \leq 1$:

$$(5-1)^2 + V_{out}^2 - 2(V_{in} - \frac{1}{2})V_{out} = 0$$

$$\Rightarrow V_{out}^2 - 2(V_{in} - \frac{1}{2})V_{out} + 16 = 0$$

$$\Rightarrow V_{out} = (V_{in} - \frac{1}{2}) \pm \frac{1}{2} \sqrt{4(V_{in} - \frac{1}{2})^2 - 16} = (V_{in} - \frac{1}{2}) \pm \sqrt{(V_{in} - \frac{1}{2})^2 - 4}$$

where we must use the - sign since M_n ohmic $\Rightarrow V_{in} - \frac{1}{2} \geq V_{out}$

$$\therefore V_{out} = (V_{in} - \frac{1}{2}) - \sqrt{(V_{in} - \frac{1}{2})^2 - 4}$$

as we need $1 \geq V_{out} \Rightarrow 1 \geq (V_{in} - \frac{1}{2}) - \sqrt{(V_{in} - \frac{1}{2})^2 - 4}$

$$\Rightarrow \sqrt{(V_{in} - \frac{1}{2})^2 - 4} \geq (V_{in} - \frac{3}{2})$$

$$\begin{aligned}
 \text{if } V_{in} \geq 3/2 &\Rightarrow (V_{in} - \frac{1}{2})^2 - 4 \geq (V_{in} - \frac{3}{2})^2 \\
 &\equiv V_{in}^2 - V_{in} + \frac{1}{4} - 4 \geq V_{in}^2 - 3V_{in} + \frac{9}{4} \\
 &\Rightarrow 2V_{in} \geq \frac{9}{4} - \frac{1}{4} + 4 = \frac{8}{4} + 4 = 6 \Rightarrow V_{in} \geq 3
 \end{aligned}$$

(which has $V_{in} \geq 3/2$)

$\therefore$ for $V_{in} \geq 3$ we have $V_{out} \leq 1$ and M_p in saturation with M_n ohmic

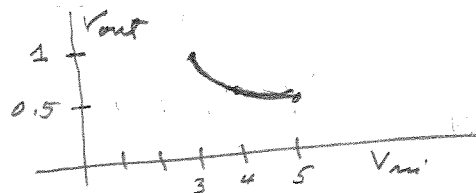
if $V_{in} \leq 3/2$ then, with $1/2 < V_{in}$ for M_n on, $\sqrt{(V_{in} - \frac{1}{2})^2 - 4}$ will not be real, so this case will not occur. Further, for $V_{in} \leq V_{TO_n} = \frac{1}{2}$, M_n is off so $I_{S_p} = I_{D_n} = 0$ implies $V_{dd} - V_{out} = 0$ since M_p has $|V_{GS}| = V_{dd} \Rightarrow M_p$ is on $\therefore$ then $V_{out} = V_{dd} = 5$ so M_p is in the ohmic region.

* $\therefore M_p$ is in saturation for $V_{in} \geq 3$

c) For $V_{in} \geq 3$ we have from part b), M_n on, ohmic and M_p saturated

$$V_{out} = (V_{in} - \frac{1}{2}) - \sqrt{(V_{in} - \frac{1}{2})^2 - 4}$$

d) We have a quadratic curve passing through

$$\begin{aligned}
 V_{in} = 5 &\Rightarrow V_{out} = 4.5 - \sqrt{(4.5)^2 - 4} = 0.469 \\
 V_{in} = 3 &\Rightarrow V_{out} = 2.5 - \sqrt{(2.5)^2 - 4} = 1 \\
 V_{in} = 4 &\Rightarrow V_{out} = 3.5 - \sqrt{(3.5)^2 - 4} = 0.628
 \end{aligned}$$


e) This is somewhat an inverter but not very good since at $V_{in} = 5$ it is not 0 and at $V_{in} = 3$ still only at 4v.

d) If we include the Early voltage the I_{D_n} for M_n ohmic becomes cubic in V_{out} , so difficult to exactly solve

$$I_{D_n} = \frac{K_P W}{2 L} (2(V_{in} - \frac{1}{2})V_{out} - V_{out}^2)(1 + \lambda V_{out})$$

but with $\lambda = 0.008$ the cubic term does not have a large effect.