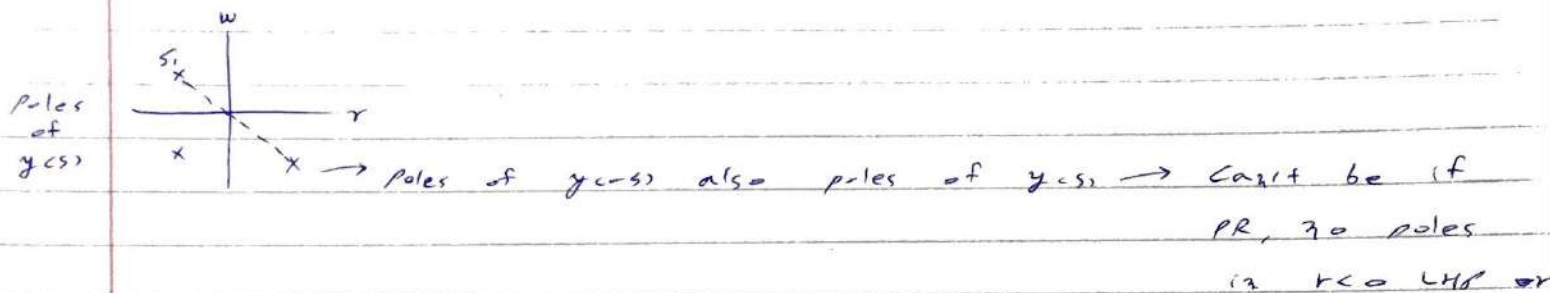


1. Scattering matrix does not exist for either the voltage or current negative impedance converters.

$y(s) = -y(s)$ if LPR driving port



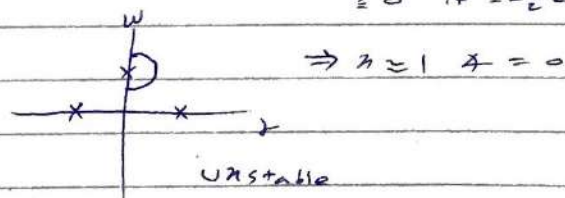
⇒ All poles were on JW axis RHP

Such a pole has a partial expansion

& near a pole $s \pm j\omega$

This is $\frac{k}{s - j\omega_0 \pm j\omega_0} = |k| e^{j\theta} / |s - j\omega_0 \pm j\omega_0| e^{j\theta} = \text{ratio } e^{j(\theta - \theta)}$

As PR $\cos \theta \geq 0$ $\cos \theta = \cos(\theta - \theta) = \cos \theta$
 ≥ 0 if $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$



$\sum \text{Residue} \left(\frac{k_1}{s - j\omega_0 \pm j\omega_0} + \frac{k_2}{s + j\omega_0 \pm j\omega_0} \right)$

$\frac{2k_1 \delta}{s^2 + \omega_0^2} \Rightarrow$ can't be negative odd f

∴ Scattering matrix does not exist for either voltage or current negative impedance converters

2-(a) $\frac{V_o(s)}{V_i(s)} = \frac{5s(s+2)}{(s+1)(s+3)}$

$Y(s) = \frac{5s(s+2)}{(s+1)(s+3)} \cdot \frac{1}{s} = \frac{5(s+2)}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3}$
 $= A(s+3) + B(s+1) = 5s + 10$
 $= As + 3A + Bs + B = 5s + 10$
 $A + B = 5 \quad 3A + B = 10 \quad A = 2.5 \quad B = 2.5$

$$Y(s) = \frac{2.5}{s+1} + \frac{2.5}{s+3} \Rightarrow y(t) = 2.5e^{-t} + 2.5e^{-3t}$$

$$y(t) = 2.5(e^{-t} + e^{-3t})$$

$$2-(b) \quad Y(s) = \frac{5s(s+2)}{(s+1)(s+3)} - 1 = 5 + \frac{-5(s+3)}{(s+1)(s+3)} = 5 + \frac{A}{s+1} + \frac{B}{s+3}$$

$$A(s+3) + B(s+1) = -10s - 15$$

$$As + Bs = -10s, \quad 3A + B = -15$$

$$A + B = -10$$

$$A = -2.5 \quad B = -7.5$$

$$\therefore Y(s) = 5 + \frac{-2.5}{s+1} + \frac{-7.5}{s+3}$$

$$y(t) = 5 - 2.5e^{-t} - 7.5e^{-3t}$$

$$2-(c) \quad \frac{V_{out}}{V_{in}}(s) = \frac{5s^2 + 10s}{s^2 + 4s + 3} = \frac{1}{\frac{(5s^2 + 10s)(s+3) + \frac{1}{5}}{10s^3 + 15s^2 + 20s + 30s + \frac{1}{5}}}$$

$$= \frac{1}{10s^3 + 15s^2 + 30s + \frac{1}{5}} = \frac{1}{10s^3 + 15s^2 + 30s + \frac{1}{5}}$$

$$= \frac{1}{s^3 + 2.5s^2 + 3s + \frac{1}{50}}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.02 & -3 & -2.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V_{in}$$

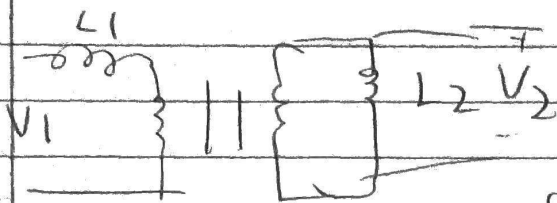
$$V_o = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\exp(A) = \begin{bmatrix} 1 & e & 1 \\ 1 & 1 & e \\ e^{-0.02} & e^{-3} & e^{-2.5} \end{bmatrix}$$

From $\exp(A)$, it can be compared with 2-(a).

3a) $L = \begin{pmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{pmatrix}$

$V = \omega L i$



$\Rightarrow (V_1 - \omega L_{11} i) T = V_2$ $\leftarrow \text{from } V_0 = T \frac{V_1}{i}$

$V_1 = \omega L_{11} i_1 + \omega L_{12} i_2$

$V_2 = \omega L_{12} i_1 + \omega L_{22} i_2$

$\Rightarrow i_2 = \frac{V_2}{\omega L_{22}} - \frac{L_{21} i_1}{L_{22}}$

$V_1 = \omega L_{11} i_1 + \omega L_{12} \left(\frac{V_2}{\omega L_{22}} - \frac{L_{21} i_1}{L_{22}} \right)$

$V_1 = \omega \left(L_{11} - \frac{L_{12} L_{21}}{L_{22}} \right) i_1 + \frac{L_{12}}{L_{22}} V_2$

$\Rightarrow V_2 = \frac{L_{22}}{L_{12}} \left(V_1 - \omega \left(L_{11} - \frac{L_{12} L_{21}}{L_{22}} \right) i_1 \right)$

$\underbrace{\qquad\qquad\qquad}_T \qquad\qquad\qquad \underbrace{\qquad\qquad\qquad}_{L_1}$

$V_2 = (V_1 - \omega L_{11} i_1) T$

$T = \frac{L_{22}}{L_{12}}, \quad L_1 = \left(L_{11} - \frac{L_{12} L_{21}}{L_{22}} \right)$

$\Rightarrow$ for a transformer $i_1 = -T i_2$ $\checkmark$ in opposite direction of i_2

$i_2 = \frac{V_2}{\omega L_2}$

$\Rightarrow i_1 = -T \left(i_2 = \frac{V_2}{\omega L_2} \right)$

$$3a) i_1 = -T i_2 = \frac{V_2}{\Delta L_{12}}$$

from

$$V_2 = \Delta L_{12} i_1 + \Delta L_{22} i_2$$

$$\Delta L_{12} i_1 = V_2 - \Delta L_{22} i_2$$

$$i_1 = \frac{V_2}{\Delta L_{12}} - \frac{\Delta L_{22}}{\Delta L_{12}} i_2$$

$$i_1 = \frac{\Delta L_{22}}{\Delta L_{12}} \left(i_2 - \frac{V_2}{\Delta L_{22}} \right)$$

$$\Rightarrow T = \frac{\Delta L_{22}}{\Delta L_{12}}, \quad L_2 = \Delta L_{22},$$

$$L_1 = \left(L_{11} - \frac{(L_{12})^2}{L_{22}} \right)$$

$$3b) k = \frac{|L_{12}|}{\sqrt{L_{11} L_{22}}} = 1 \Rightarrow |L_{12}| = \sqrt{L_{11} L_{22}}$$

$$\Rightarrow (L_{12})^2 = L_{11} L_{22}$$

$$\Rightarrow \frac{(L_{12})^2}{L_{22}} = L_{11}$$

$$\text{so } L_1 = L_{11} - \frac{(L_{12})^2}{L_{22}} \quad \leftarrow \text{but } L_{11} - \frac{(L_{12})^2}{L_{22}}$$

$$\Rightarrow L_1 = 0 \leftarrow \text{so } L_1 \text{ is removed}$$