

HW5

Problem 1

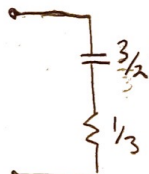
(a) $S_1 = \frac{s-1}{s+2}$ and $S_2 = \frac{s+3}{s+4}$ we know that $S = (1+y)^{-1}(1-y)$

$\Rightarrow S(1+y) = (1-y) \Rightarrow s + Sy = 1 - y \Rightarrow s - 1 = -Sy - y \Rightarrow s - 1 = -(s+1)y$

$\Rightarrow -\frac{s-1}{s+1} = y \quad (*)$

So by $(*)$, $y_1 = \frac{\left(\frac{s-1}{s+2}\right) - 1}{\left(\frac{s-1}{s+2}\right) + 1} = -\frac{\left(\frac{s-1-s-2}{s+2}\right)}{\left(\frac{s-1+s+2}{s+2}\right)} = +\frac{s+3}{2s+1}$

$= \frac{1}{\frac{2}{3}s + \frac{1}{3}} \Rightarrow$



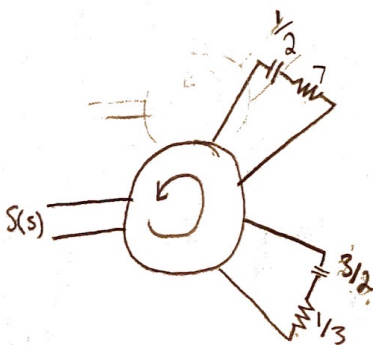
reciprocal

Using $(*)$ for S_2 , we have $y_2 = -\frac{\left(\frac{s+3}{s+4}\right) - 1}{\left(\frac{s+3}{s+4}\right) + 1} = -\frac{\frac{s+3-s-4}{s+4}}{\frac{s+3+s+4}{s+4}} = -\frac{-1}{2s+7}$

$= \frac{1}{2s+7} \Rightarrow$



Using a circulator, we can create the final circuit, observing that $S = S_1(s)S_2(s)$:

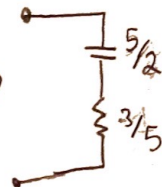


$$(b) \quad S_1 = \frac{s-1}{s+4}, \quad S_2 = \frac{s+3}{s+2}$$

Again, we will apply (*) to obtain the admittances corresponding with S_1 and S_2 :

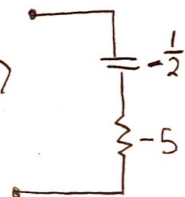
For S_1 :

$$-\frac{\left(\frac{s-1}{s+4}\right)-1}{\left(\frac{s-1}{s+4}\right)+1} = -\frac{\frac{s-1-s-4}{s+4}}{\frac{s-1+s+4}{s+4}} = -\frac{\frac{-s-5}{s+4}}{\frac{s+3}{s+4}} = \frac{s+5}{2s+3} \Rightarrow \frac{1}{\frac{2s}{5} + \frac{3}{5}} \Rightarrow$$

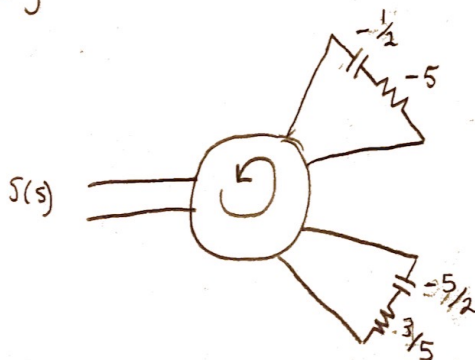


For S_2 :

$$-\frac{\left(\frac{s+3}{s+2}\right)-1}{\left(\frac{s+3}{s+2}\right)+1} = -\frac{\frac{s+3-s-2}{s+2}}{\frac{s+3+s+2}{s+2}} = -\frac{\frac{1}{s+2}}{\frac{s+5}{s+2}} = \frac{-1}{2s+5} = \frac{1}{-2s-5} \Rightarrow$$



Using the circulator, we create the final circuit for $S(s)$:

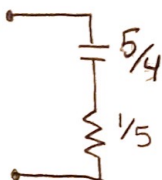


$$(C) \quad S_1 = \frac{s-1}{s+1.5} \quad S_2 = \frac{(s+1.5)(s+3)}{(s+2)(s+4)}$$

$\uparrow \quad \quad \uparrow$
 $s_3 \quad s_4$

Applying (B), we obtain the admittances for S_1 , S_3 , and S_4

For S_1 :

$$-\frac{\left(\frac{s-1}{s+1.5}\right)-1}{\left(\frac{s-1}{s+1.5}\right)+1} = -\frac{\left(\frac{s-1-s-1.5}{s+1.5}\right)}{\left(\frac{s-1+s+1.5}{s+1.5}\right)} = \frac{2.5}{2s+0.5} = \frac{1}{\frac{4}{5}s+\frac{1}{5}} \Rightarrow$$


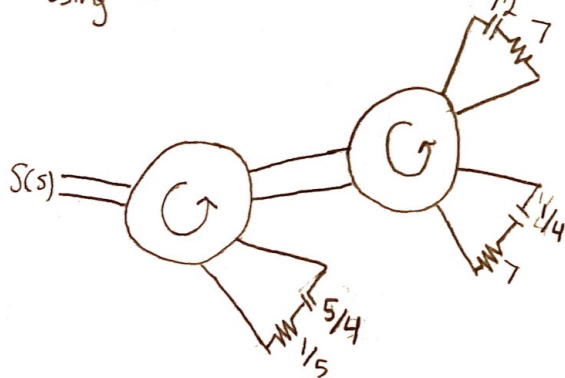
For S_3 :

$$-\frac{\left(\frac{s+1.5}{s+2}\right)-1}{\left(\frac{s+1.5}{s+2}\right)+1} = -\frac{\left(\frac{s+1.5-s-2}{s+2}\right)}{\left(\frac{s+1.5+s+2}{s+2}\right)} = \frac{0.5}{2s+3.5} = \frac{1}{4s+7} \Rightarrow$$


For S_4 :

$$-\frac{\left(\frac{s+3}{s+4}\right)-1}{\left(\frac{s+3}{s+4}\right)+1} = -\frac{\left(\frac{s+3-s-4}{s+4}\right)}{\left(\frac{s+3+s+4}{s+4}\right)} = \frac{-1}{2s+7} \Rightarrow$$


Using a circulator connected to another circulator, we obtain $S(s)$:



(d) In the first two realizations from (a)/(b), we only have one circulator, whereas in part (c), having S_2 be the product of two rational functions generates another circulator. We can also note a notable difference that comes about from part (b): the resistance and capacitances are negative in this circuit, which is not the case in either of the other circuits.

Problem 2

(a) FT: $G(\underbrace{j2\pi f}_\omega) = \exp(-\pi f^2)$

$$\Rightarrow G(\underbrace{j\omega}_s) = \exp\left(-\pi\left(\frac{\omega}{2\pi}\right)^2\right) = \exp\left(-\frac{\omega^2}{4\pi}\right) = \exp\left(-\frac{\left(\frac{j}{j}\omega\right)^2}{4\pi}\right) = \exp\left(-\frac{(j\omega)^2}{j^2 4\pi}\right)$$

$$\Rightarrow G(s) = \exp\left(-\frac{s^2}{j^2 4\pi}\right) = \exp\left(\frac{s^2}{4\pi}\right)$$

So $G(s) = \exp\left(\frac{s^2}{4\pi}\right)$

(c) Some uses I found of Gaussian functions on the web;

- 1) Gaussian functions are used in the field of wavelet transforms and pattern recognition.
- 2) Gaussian functions can be used for classification applications, which can be performed using a CMOS analogue circuit.