

ENEE 605 HW 3

1a) V, Z, S

$$Y_C \Rightarrow i_1 = (\Delta C + 1/R)(V_1 - V_2)$$

$$i_2 = -(\Delta C + 1/R)(V_1 - V_2)$$

$$= Y_C = \begin{bmatrix} \Delta C + 1/R & -(\Delta C + 1/R) \\ -(\Delta C + 1/R) & \Delta C + 1/R \end{bmatrix} \quad Y_g = \begin{bmatrix} 0 & 0 \\ 0 & g \end{bmatrix}$$

$$Y = \begin{bmatrix} \Delta C + 1/R & -(\Delta C + 1/R) \\ -(\Delta C + 1/R) & \Delta C + 1/R + g \end{bmatrix}$$

$$Z = Y^{-1} \quad \det(Y) = (\Delta C + 1/R)(\Delta C + 1/R + g) - (\Delta C + 1/R)^2$$

$$\det(Y) = (\Delta C)^2 + 2\Delta C(1/R) + \Delta C g + (1/R)^2 + (1/R)g - (\Delta C^2 - 2\Delta C(1/R) - (1/R)^2)$$

$$\det(Y) = (\Delta C + 1/R)g$$

$$Y^{-1} = \frac{1}{\Delta C g + g/R} \begin{bmatrix} \Delta C + 1/R + g & \Delta C + 1/R \\ \Delta C + 1/R & \Delta C + 1/R \end{bmatrix} = Z$$

$$S = (I_m - Y)^{-1} (Z_m + Y) \quad Z = \begin{bmatrix} \frac{\Delta C + 1/R + g}{(\Delta C + 1/R)g} & 1/g \\ 1/g & 1/g \end{bmatrix}$$

$$I_m - Y = \begin{bmatrix} 1 - (\Delta C + 1/R) & \Delta C + 1/R \\ \Delta C + 1/R & 1 - (\Delta C + 1/R + g) \end{bmatrix}$$

$$\det(I_m - Y) = (1 - \Delta C - 1/R)(1 - \Delta C - 1/R - g) - (\Delta C + 1/R)^2$$

$$= (1 - \Delta C - 1/R - g) + (1 - g)(-\Delta C - 1/R) + (-\Delta C - 1/R)^2 - (\Delta C + 1/R)^2$$

$$= \Delta C g + g/R - 2\Delta C - 2/R - g + 1$$

$$1a) (Z_m - Y)^{-1} = \frac{1}{1 - 2\alpha(-\frac{1}{R} + \alpha(g + \frac{1}{R} - g))} \begin{bmatrix} 1 - \alpha - \frac{1}{R} & \alpha(-\frac{1}{R} + g) \\ -\alpha(-\frac{1}{R} + g) & 1 - \alpha - \frac{1}{R} - g \end{bmatrix}$$

$$(Z_m - Y)^{-1} (Z_m + Y)$$

$$= \frac{1}{\det(\cdot)} \begin{bmatrix} 1 - \alpha - \frac{1}{R} & \alpha(-\frac{1}{R} + g) \\ -\alpha(-\frac{1}{R} + g) & 1 - \alpha - \frac{1}{R} - g \end{bmatrix} \begin{bmatrix} \alpha(-\frac{1}{R} + g) & -\alpha - \frac{1}{R} \\ -\alpha - \frac{1}{R} & \alpha(-\frac{1}{R} + g) \end{bmatrix}$$

$$= \frac{1}{\det(\cdot)} \begin{bmatrix} \alpha(-\frac{1}{R} + g) + 2(-(\alpha)^2 - (\frac{1}{R})^2) & -\alpha - \frac{1}{R} + g(\alpha(-\frac{1}{R} + g) + 2(\alpha(-\frac{1}{R} + g))^2) \\ (\alpha(-\frac{1}{R} + g))^2 + (1-g)(-\alpha - \frac{1}{R}) & -2(\alpha^2 + (\frac{1}{R})^2) + (1-g)(\alpha(-\frac{1}{R} + g) + (1-g)(\frac{1}{R} + g)) \end{bmatrix}$$

$$\text{SE} \left| \frac{1}{1 - 2\alpha(-\frac{1}{R} + \alpha(g + \frac{1}{R} - g))} \begin{bmatrix} \alpha(-\frac{1}{R} + g) + 2(-(\alpha)^2 - \frac{1}{R}^2) & -\alpha - \frac{1}{R} + g(\alpha(-\frac{1}{R} + g) + 2(\alpha(-\frac{1}{R} + g))^2) \\ (\alpha(-\frac{1}{R} + g))^2 + (1-g)(-\alpha - \frac{1}{R}) & -2(\alpha^2 + \frac{1}{R}^2) + (1-g)(\alpha(-\frac{1}{R} + g) + (1-g)(\frac{1}{R} + g)) \end{bmatrix} \right|$$

$$2b) \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad \begin{matrix} i_1 = I_s \\ V_2 = V_{out} \end{matrix}$$

$$\begin{bmatrix} I_s \\ 0 \end{bmatrix} = \begin{bmatrix} sC + 1/R & -sC - 1/R \\ -sC - 1/R & sC + 1/R + g \end{bmatrix} \begin{bmatrix} V_1 \\ V_{out} \end{bmatrix}$$

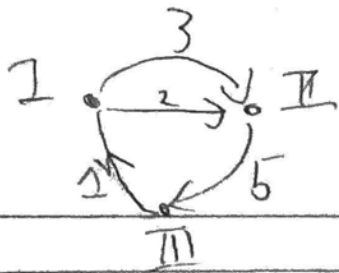
$$I_s = 0$$

$$\begin{bmatrix} I_s \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & g \\ -sC - 1/R & sC + 1/R + g \end{bmatrix} \begin{bmatrix} V_1 \\ V_{out} \end{bmatrix}$$

$$\Rightarrow -I_s = -g V_{out}$$

$$\boxed{\frac{V_{out}}{I_s} = \frac{1}{g}}$$

1c)



Use tree 1, 2

Note $V_4 = V_5$

$$A \quad V \quad B \quad i$$

$$1 \quad 2 \quad 3 \quad 5$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} i$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix} \quad \tilde{C} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

$$Ae^T - B\tilde{C}^T$$

$$\begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & -1 & 0 \\ -1 & -1 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \end{bmatrix} = \begin{bmatrix} i_5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

1d) $\Rightarrow -i_5 = i_5$ Note $V_{out} = V_5 = V_1 + V_2$

$$2V_2 + i_3 - i_5 = 0$$

$$V_2/R - i_3 = 0 \Rightarrow V_2 = Ri_3$$

$$-gV_1 - gV_2 - i_5 = 0$$

$$\Rightarrow -g(V_1 + V_2) - i_5 = 0$$

$$-g(V_1 + V_2) + i_5 = 0$$

$$= g(V_1 + V_2) = -i_5$$

$$(V_1 + V_2) = +i_5/g$$

$$V_{out} = +i_5/g$$

$$\boxed{V_{out} = \frac{i_5}{g}}$$

2a) $\ddot{x} = -E(x^2 + (\dot{x})^2 - r^2)x - \omega_0^2 x$

2b) for $E_3 = 10$ and $x(0) = 5$, $y(0) = 5$
it does not converge

- following in order

$E_1 = 0.0001$, $x(0) = y(0) = 0.5$

$E_1 = 0.0001$, $x(0) = y(0) = 5$

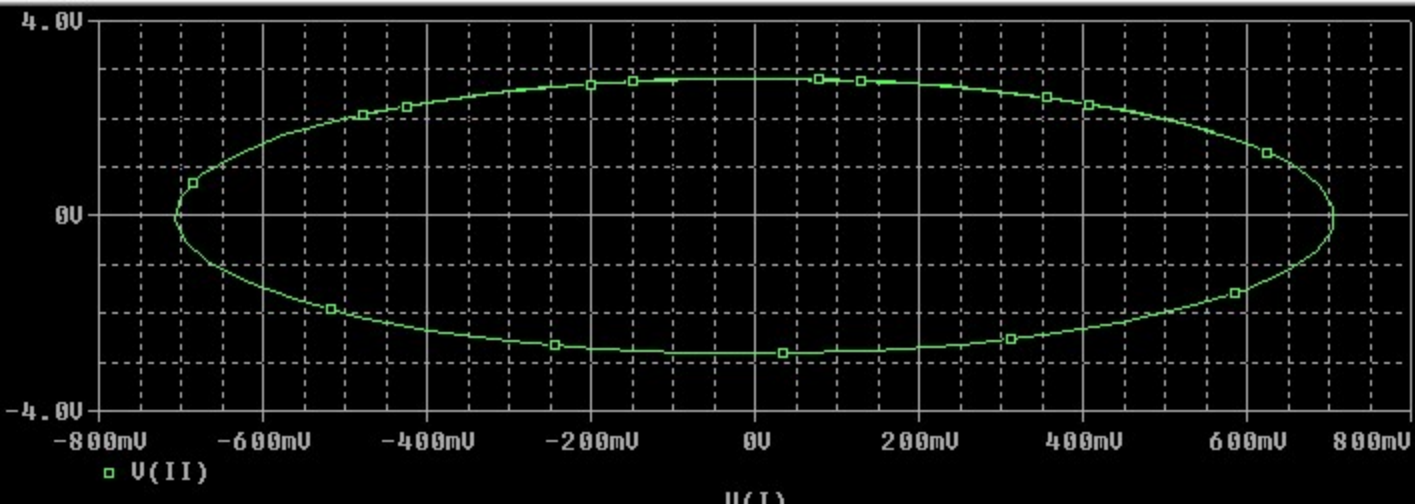
$E_2 = 1$, $x(0) = y(0) = 0.5$

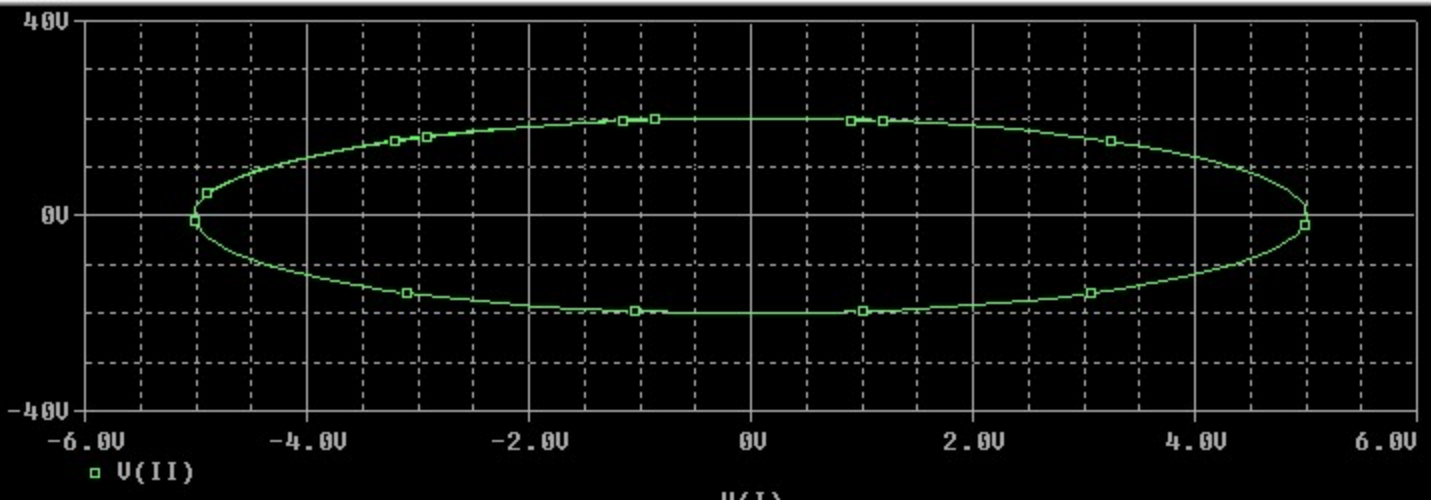
$E_2 = 1$, $x(0) = y(0) = 5$

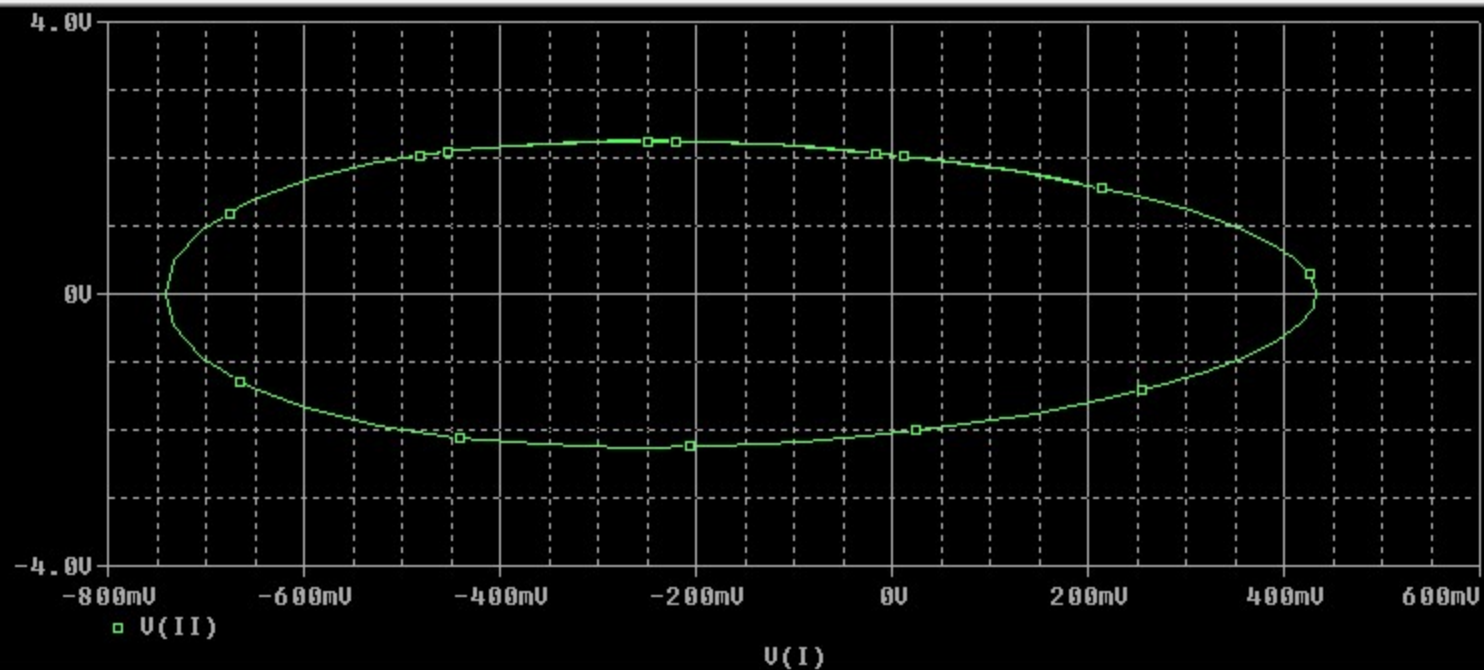
$E_3 = 10$, $x(0) = y(0) = 0.5$

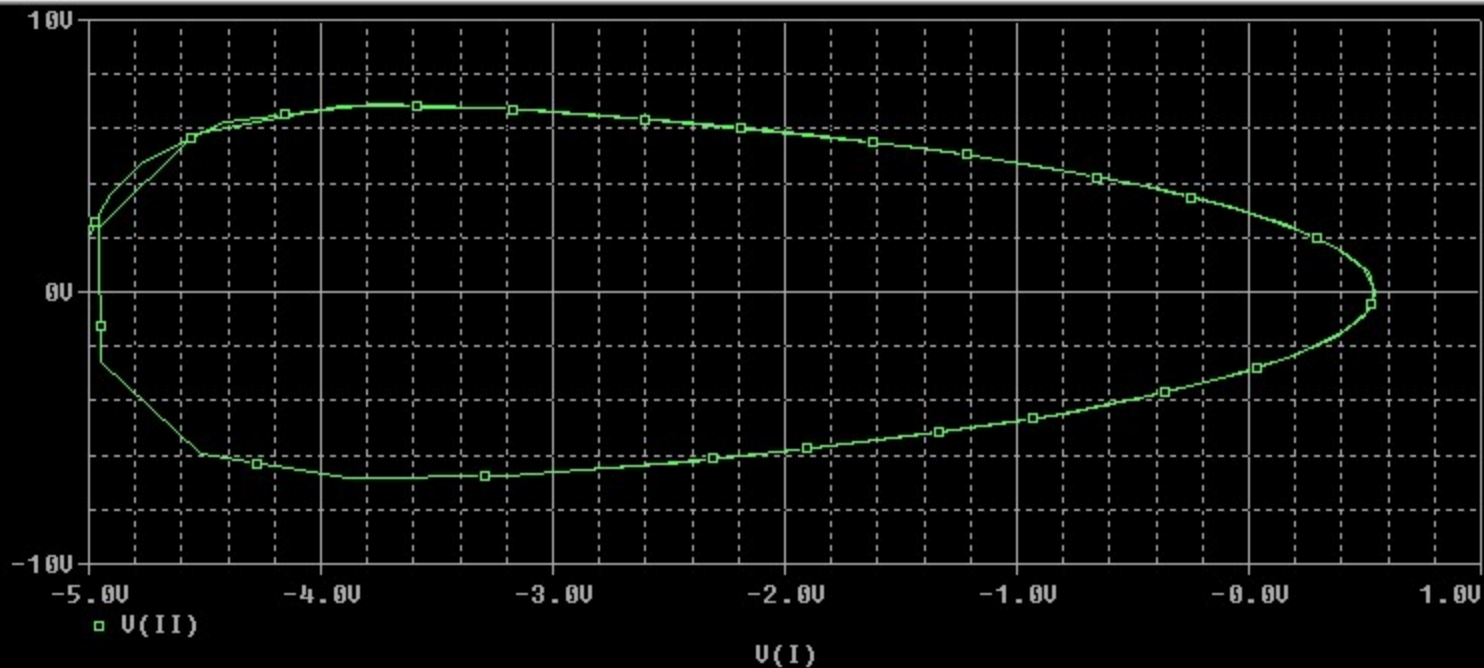
$E_3 = 10$, $x(0) = y(0) = 5$

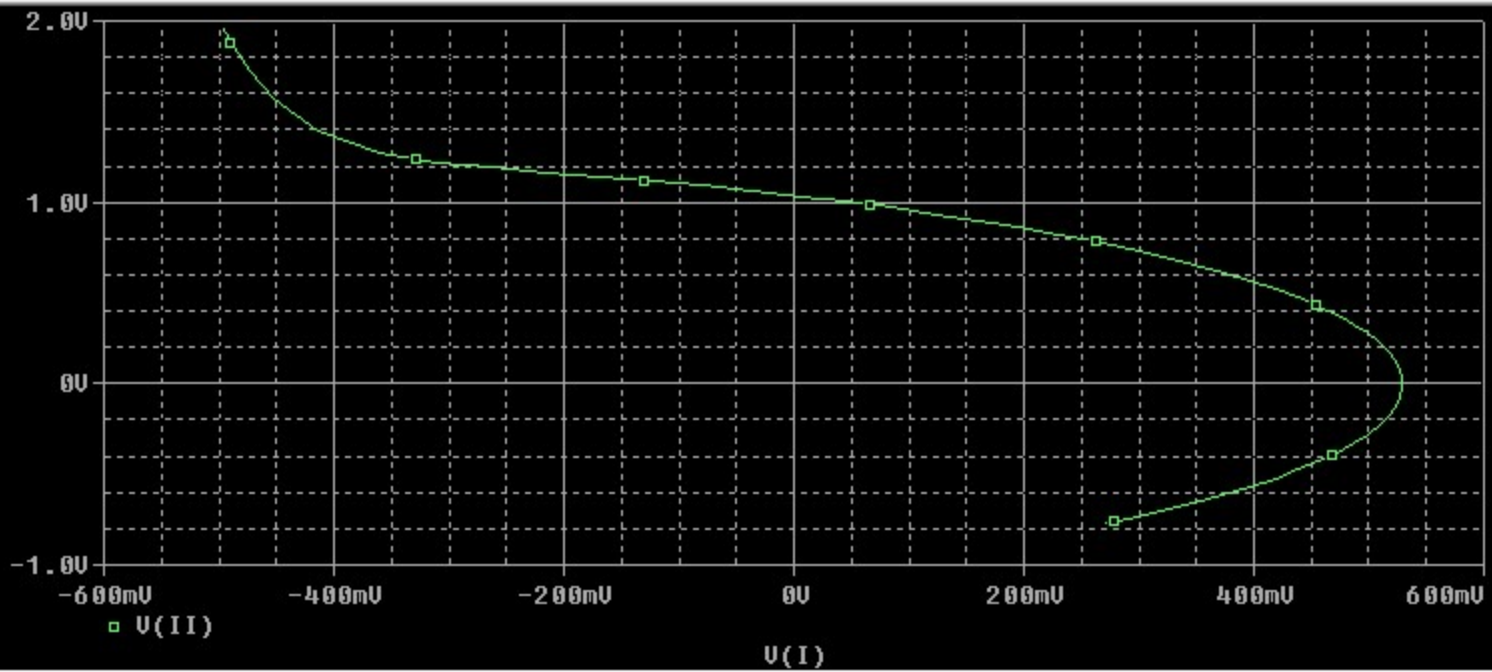
(last one would not converge / simulate











$$x_{in} = E(V_1^2 + V_2^2 - r^2)$$

