

NAME: AYOO LUWA AJIBOYE

COURSE: ENEE 610 [ELECTRICAL NETWORK THEORY]

SEMESTER: FALL 2020

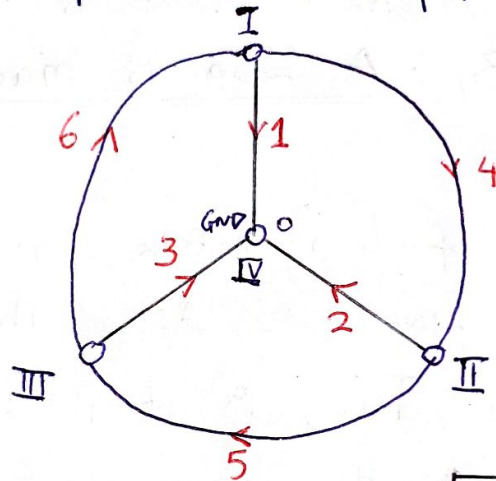
PROFESSOR: DR. ROBERT WAYNE NEWCOMB

DATE DUE: SEPTEMBER 15th, 2020 (TUESDAY) by 6pm

TITLE: HOMEWORK 1 SOLUTIONS

QUESTION 1 SOLUTION:

Considering The 3-Spoke Wheel Graph Given Below:



A Tree Refers To A Connected Subgraph of a graph consisting of ALL the NODES AND WITH NO LOOPS

No. of Branches, $b = 6$

No. of Nodes, $n+1 = 4$
 $\therefore n = 3$

Hence, No. of TWIGS = $n = 3$

AND No. of LINKS = $b - n$
 $= 6 - 3$
 $= 3$

Note;

→ A Twig(s) Refers To The Branches Appearing On A Selected Tree

→ LINKS Refer To The Branches NOT Appearing On The Corresponding Tree

Also, FOR A SELECTED TREE:

No. of f-cut-sets = No. of Twigs = n

AND No. of f-loops = No. of LINKS = $b - n$

where
f-cut-set → Fundamental cut-set

f-loop → Fundamental loop

• f-cut-set ties to the CUT-SET MATRIX
• f-loop Ties to the TIE-SET MATRIX

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A.) To determine The **Cut-Set Matrix** for a **Selected Tree**, i.e Q_f

We remove a tree branch b_k from The Tree Thereby Separating the Tree into EXACTLY Two (2) Unconnected Parts

→ Then, We Consider The LINKS Connecting The Two (2) Parts In Addition to the tree branch b_k As An f-Cut-set

The Fundamental Cut-Set (f-cut-set) Corresponding To A particular Tree is given By The Matrix:

$Q_f = [Q_{ij}]$

where $i = \text{f-cut-set} = \text{rows}$
 $j = \text{Branches} = \text{Columns}$

Q_f is an $n \times b$ order Matrix where $n = \text{Twig} = \text{f-cut-set}$
 $b = \text{branch}$

→ kindly note That The Cut-Set Orientation is determined by The Orientation of The Defining TWIG OR TREE BRANCH

B.) To determine the Tie-Set Matrix for A Selected Tree, i.e B_f

Note That Tie Set Matrix is Also known As FUNDAMENTAL loop OR f-loop Matrix denoted by B_f . A loop is made up of A Tree AND A LINK.

→ Hence, To determine The f-loop matrix for A Selected Tree, We replace each of the LINKS one After The Other. When You Replace The LINK on The Tree, A Loop is Created.

→ Then We Consider All of The Branches Present in the Loop And their Orientations relative To the loop Orientation.

Please Note THAT THE LOOP ORIENTATION IS DETERMINED BY ITS DEFINING LINK.

→ The FUNDAMENTAL - loop i.e f-loop Tie-Set Matrix Corresponding To A particular Tree is Given by:

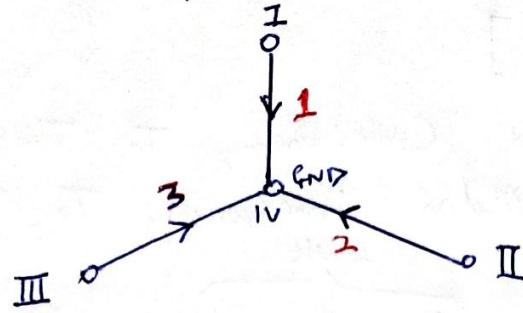
$B_f = [B_{ij}]$

where $i = \text{f-loop} = \text{rows}$
 $j = \text{Branches} = \text{Columns}$

Note; $\text{No. of f-loops} = \text{No. of links} = b - n$

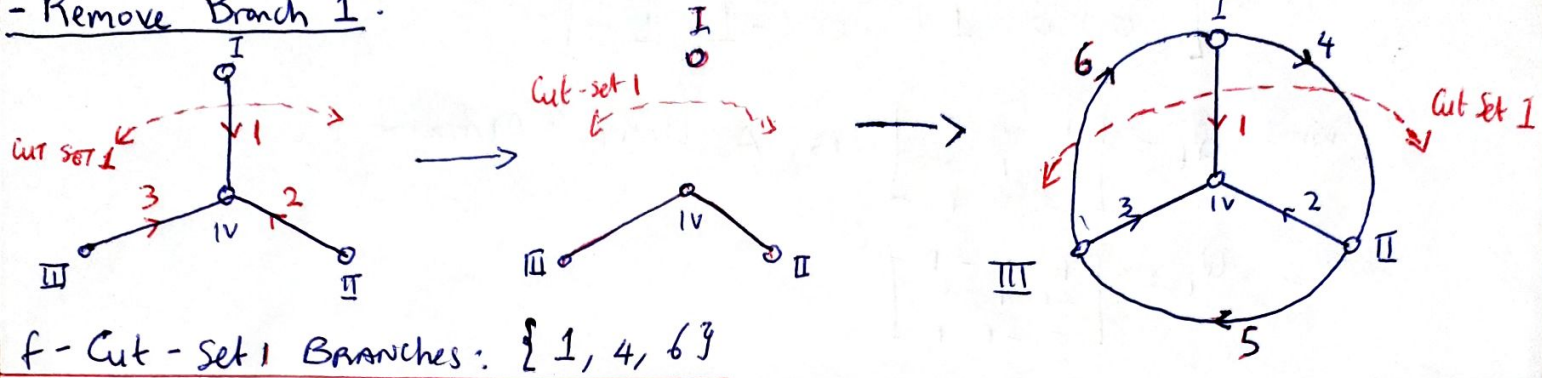
B_f is an $(b - n) \times b$ Order Matrix

1a.) USING BRANCHES 1, 2, 3 AS A TREE GIVES:



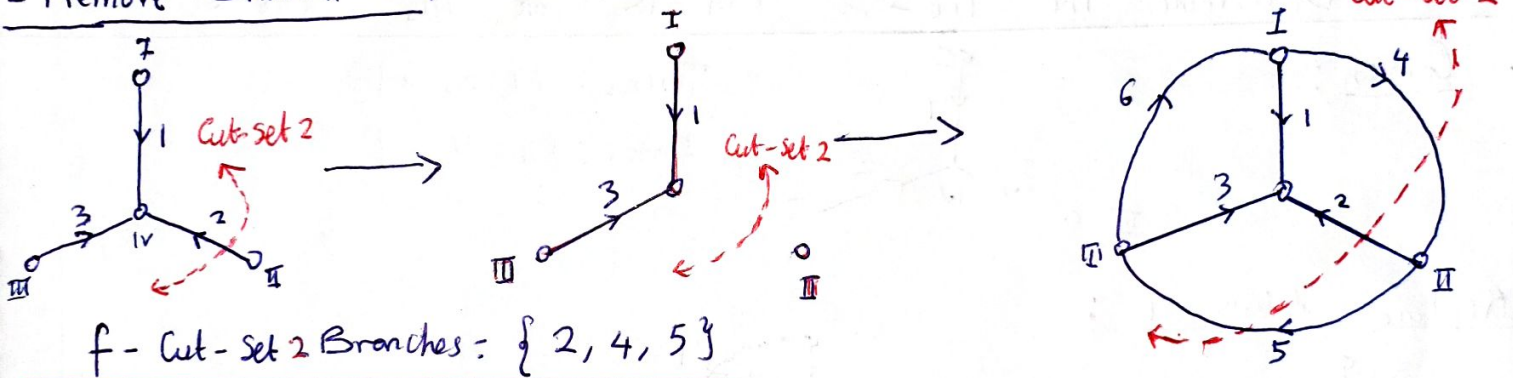
i.) To Determine the Cut-set Matrix i.e Q_f ;

- Remove Branch 1:



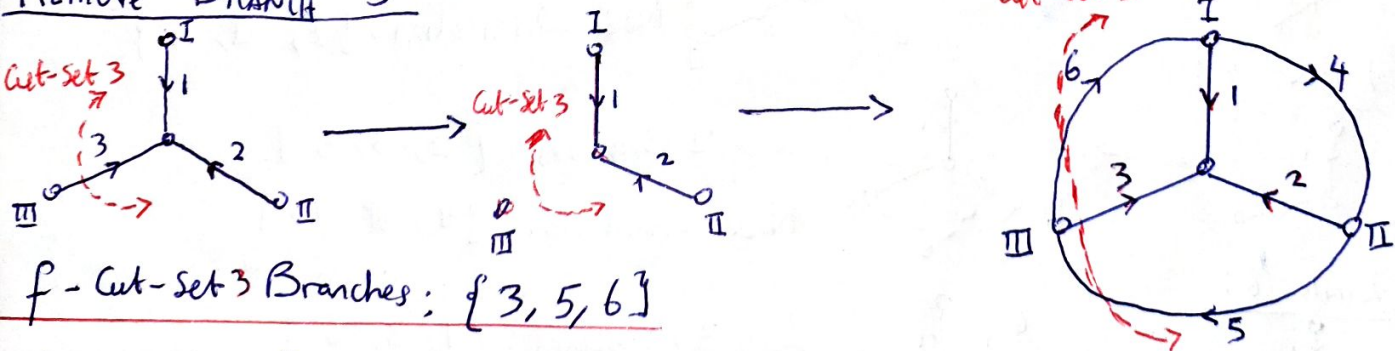
f - Cut - set 1 Branches: $\{1, 4, 6\}$

- Remove BRANCH 2:



f - Cut - set 2 Branches: $\{2, 4, 5\}$

Remove BRANCH 3:



f - Cut - set 3 Branches: $\{3, 5, 6\}$

Hence, The f - Cut - Set is:

$$Q_f = \begin{matrix} \text{f-cut-set} \\ \downarrow \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{matrix} \begin{matrix} \text{BRANCHES} \\ \rightarrow \end{matrix} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$$

Q_f is A 3×6 MATRIX

Re-writing The Q_f in The Form:

$$Q_f = [Q_t \quad Q_l]$$

where Q_t is the $n \times n$ Matrix Corresponding To The Twigs
and Q_l is the $n \times (b-n)$ Matrix Corresponding To The Links.

Here;

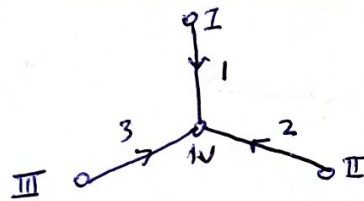
$$Q_f = \begin{matrix} \text{f-cut-set} \\ \downarrow \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{matrix} \begin{matrix} \text{Twig} & & \text{Link} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \end{matrix}$$

where $Q_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is A UNIT MATRIX

$$\text{and } Q_l = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

ii.) To Determine The Tie-Set MATRIX For This Tree:

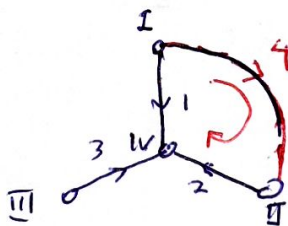
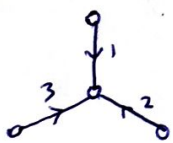
- Given:



Twigs: {1, 2, 3}

Links: {4, 5, 6}

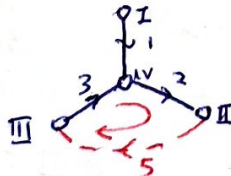
- Replace Link 4:



Loop 1 Branches: {1, 2, 4}

Node Orientation: {I, II, IV}

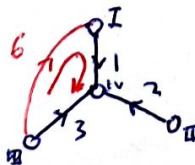
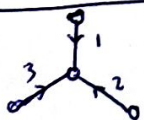
Replace Link 5:



Loop 2 Branches: {2, 3, 5}

Node Orientation: {II, III, IV}

Replace Link 6:



Loop 3 Branches: {1, 3, 6}

Node Orientation: {I, IV, III}

The Tie-Set MATRIX:

Branches →

$$\therefore B_f = \begin{matrix} \text{f-Loops} \\ \downarrow \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{matrix} \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

Re-writing B_f in the Form: $B_f = [B_t \ B_c]$

where B_t is the $(b-n) \times n$ Submatrix Corresponding To The Twigs

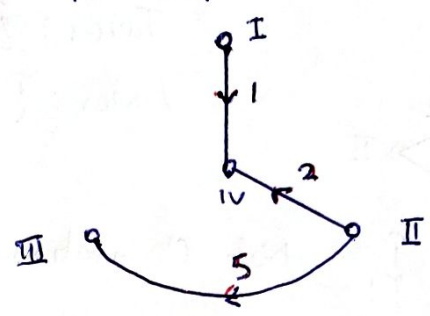
B_c is the $(b-n) \times (b-n)$ Square Submatrix Corresponding To The Links.

Therefore,

$$B_f = \begin{matrix} \text{f-loops} \\ \downarrow \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{matrix} \begin{matrix} \text{TWIG} & \text{LINK} \\ \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

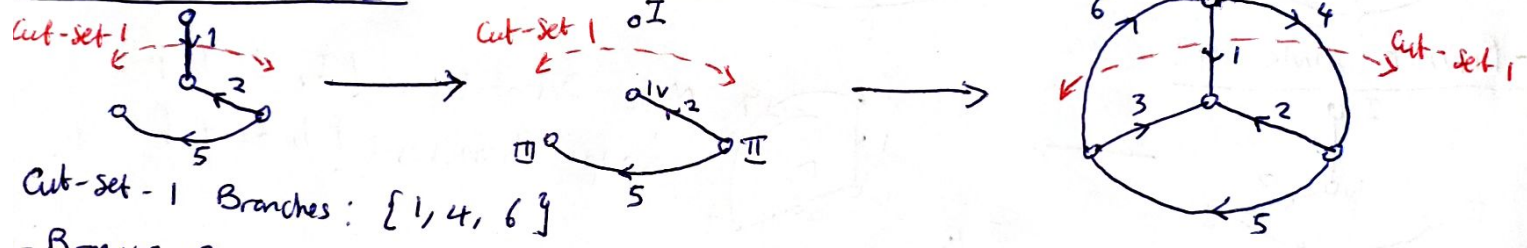
where $B_t = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$ AND $B_c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \text{UNIT MATRIX}$

1b.) USING BRANCHES 1, 2, 5 AS A TREE TO DETERMINE BOTH THE CUT-SET AND TIE-SET MATRICES GIVE:

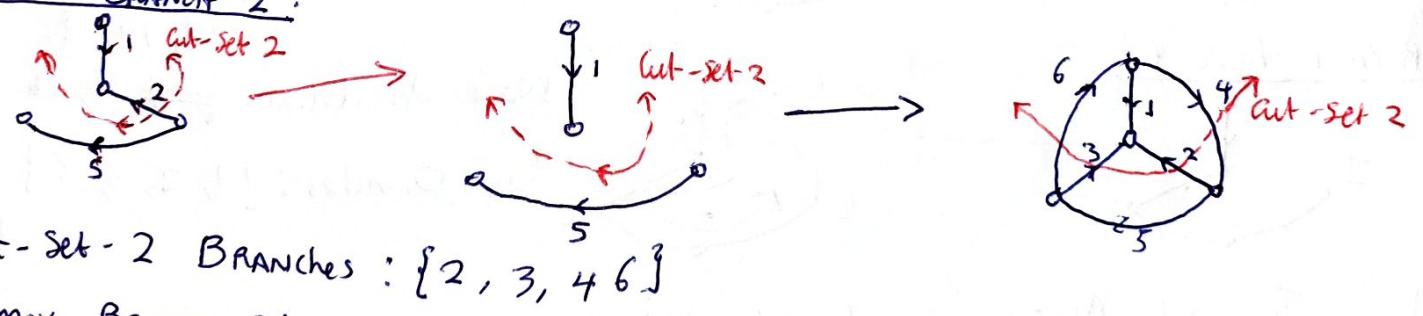


i.) To Determine The Cut-set Q_f :

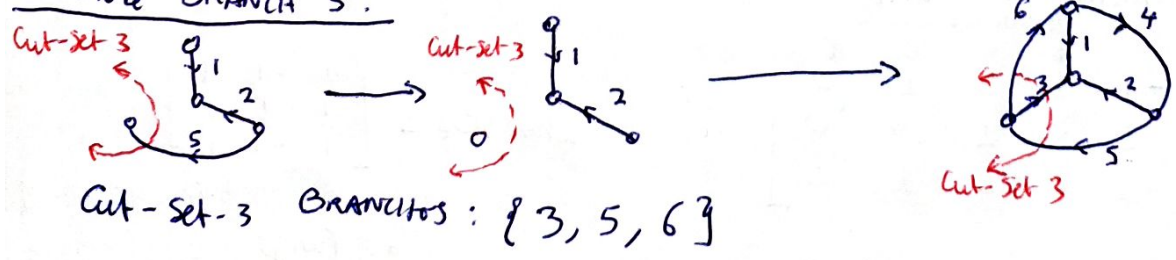
- Remove BRANCH 1:



- Remove BRANCH 2:



- Remove BRANCH 5:



Hence; The f-cut-set Matrix Q_f is Given By:

$$Q_f = \begin{matrix} \text{f-cut-set} \\ \downarrow \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{matrix} \begin{matrix} \text{Branches} \\ \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & -1 \end{bmatrix} \end{matrix}$$

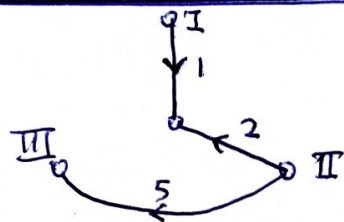
In The Form $Q_f = [Q_t \quad Q_l]$
 where Q_t is the $n \times n$ square Matrix Corresponding To Twigs
 Q_l is the $n \times (b-n)$ Matrix Corresponding To Links.

$\therefore Q_f =$ f-cut-set $\begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \left[\begin{array}{ccc|ccc} & \text{Twigs} & & \text{Links} & & \\ & 1 & 2 & 5 & 3 & 4 & 6 \\ \hline 1 & 1 & 0 & 0 & 0 & 1 & -1 \\ 2 & 0 & 1 & 0 & 1 & -1 & 1 \\ 3 & 0 & 0 & 1 & -1 & 0 & -1 \end{array} \right]$

where $Q_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \text{UNIT MATRIX}$ And $Q_l = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 0 & -1 \end{bmatrix}$

ii.) To Determine The Tie-Set MATRIX B_f

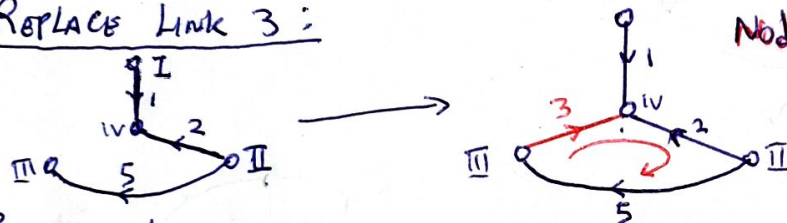
$\rightarrow$ Given :



Twigs : {1, 2, 5}

Links : {3, 4, 6}

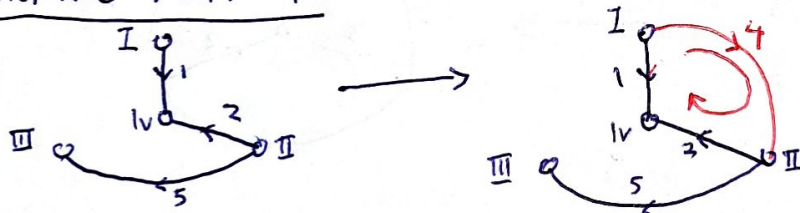
- REPLACE Link 3 :



Node Orientations : {II, III, IV}

Loop Branches : {2, 3, 5}

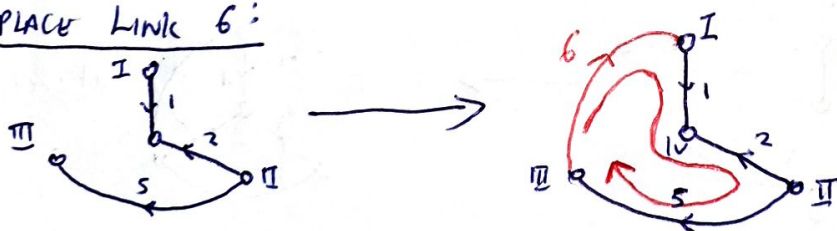
- REPLACE LINK 4 :



Node Orientations : {I, II, IV}

Loop Branches : {1, 2, 4}

- REPLACE LINK 6 :



Node Orientation : ~~{I, II, III}~~
~~{I, IV, II, III}~~

Loop Branches : {1, 2, 5, 6}

Hence, Tie-Set MATRIX IS GIVEN AS :

$B_f =$ f-loops $\begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \left[\begin{array}{cccccc} & \text{Branches} & & & & \\ & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & -1 & 1 & 0 & 1 & 0 \\ 2 & -1 & 1 & 0 & 1 & 0 & 0 \\ 3 & 1 & -1 & 0 & 0 & 1 & 1 \end{array} \right]$

where $B_t = \begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$

$(b-n) \times n$ Matrix Corresponding To Twigs

Re-Writing As $B_f = [B_t \quad B_l]$

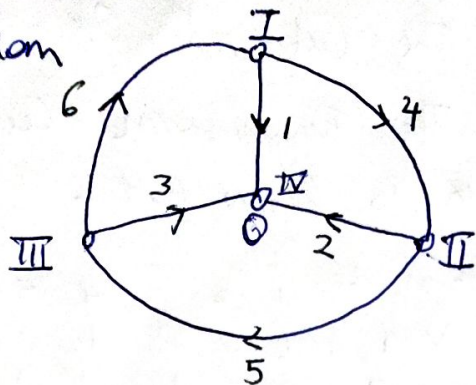
$B_f =$ f-loops $\begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \left[\begin{array}{ccc|ccc} & \text{Twigs} & & \text{Links} & & \\ & 1 & 2 & 5 & 3 & 4 & 6 \\ \hline 1 & 0 & -1 & 1 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 & 0 & 1 & 0 \\ 3 & 1 & -1 & 1 & 0 & 0 & 1 \end{array} \right]$

AND

$B_l = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \text{UNIT MATRIX}$

$(b-n) \times (b-n)$ Square Matrix Corresponding To The Links

(c.) From



no. of nodes = $n+1 = 4$

Branches = $b = 6$

To get the Complete Incidence Matrix A_a where $A_a = [a_{ij}]$

A_a is of order $(n+1) \times b$

In $A_a = [a_{ij}] \rightarrow A_a$ is a 4×6 matrix

i = rows = nodes

j = columns = branches

$$\therefore A_a = \begin{matrix} \text{Nodes} \downarrow \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \end{matrix} \begin{matrix} \text{Branches} \rightarrow \\ \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \\ -1 & -1 & -1 & 0 & 0 & 0 \end{bmatrix} \end{matrix} \end{matrix}$$

Now, The Reduced Incidence Matrix with the Ground Node i.e. Node 4 or last row eliminated

$A = [a_{ij}]$

A is of order $n \times b = 3 \times 6$ matrix

this gives:

$$A = \begin{matrix} \text{Nodes} \downarrow \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \end{matrix} \begin{matrix} \text{Branches} \rightarrow \\ \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix} \end{matrix} \end{matrix}$$

No. of Trees = $\text{Det } [A A^T]$ Page 7

$A^T \rightarrow$ Switch Rows And Columns

$\therefore A^T$ is a 6×3 matrix

$$A^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$A A^T = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$A A^T = 3 \times (6 \times 6) \times 3 = 3 \times 3$ matrix

$$A A^T = \begin{bmatrix} (1 \times 1) + (1 \times 1) + (-1 \times -1) & (1 \times -1) & (1 \times -1) \\ (-1 \times 1) & (1 \times 1) + (-1 \times -1) + (1 \times 1) & (1 \times -1) \\ (1 \times -1) & (-1 \times 1) & (1 \times 1) + (-1 \times -1) + (1 \times 1) \end{bmatrix}$$

$$A A^T = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

For A GRAPH WITH 6 Branches And 4 Nodes, This seems to be consistent.

Hence,

No. of Trees = $\text{det } (A A^T)$

$$= \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}$$

$$= +3(3 \times 3 - (-1 \times -1)) - (-1)(-1 \times 3 - (-1 \times -1)) + (-1)((-1 \times -1) - (-3 \times 3))$$

$$= 3(9-1) + 1(-3-1) - 1(1+3) = 3(8) + 1(-4) - 1(4) = 24 - 4 - 4$$

No. of Trees = 16

TREE with Branches {1, 2, 3}

TREE with Branches {1, 2, 5}

i.) Cutset: $Q_f = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$

Cutset: $Q_f = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{bmatrix}$

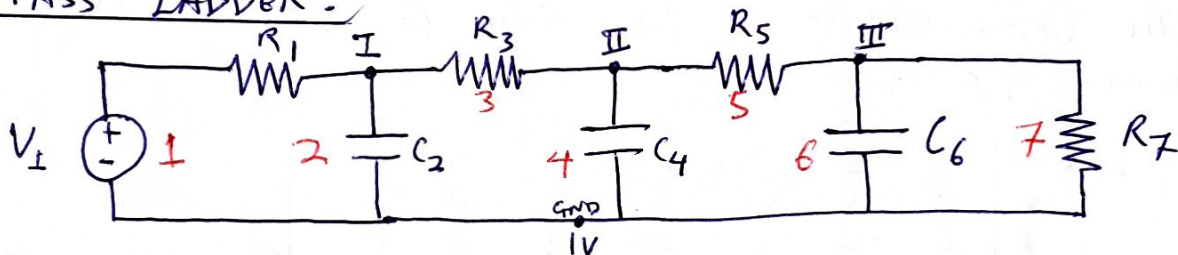
ii.) Tie-Set: $B_f = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{bmatrix}$

Tie-Set: $B_f = \begin{bmatrix} 0 & -1 & 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 \end{bmatrix}$

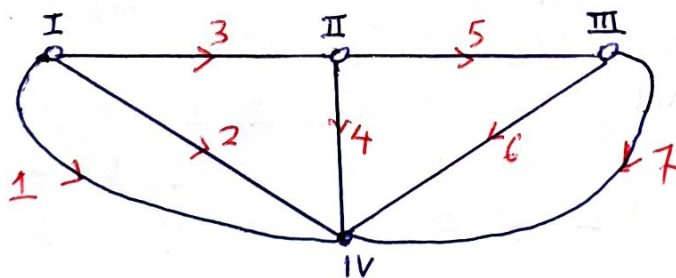
$B_f = \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$

QUESTION 2 SOLUTIONS

LOWPASS LADDER:



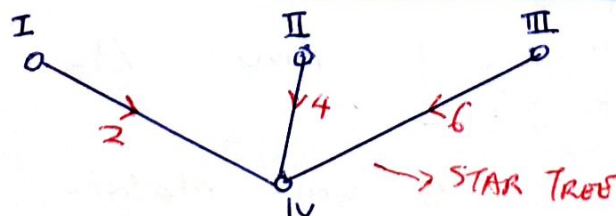
a.) GRAPH OF THE NETWORK:



Branches 1 has both V_1 AND R_1

→ ORIENT THE GRAPH TO THE RIGHT AND DOWN AS INSTRUCTED

b.) USING THE CAPACITOR BRANCHES AS A TREE



$b = 7$ branches

$n+1 = 4$ nodes

$n = 3$ twigs

$b - n = 4$ links

→ STAR TREE

→ Note That The Tree is A Star Tree Because All The Twigs Are INCIDENT AT A Common Node IV And They All have A Terminal Node i.e. A Node with Only One Branch Incident

RECALL KCL Equation:

$$A i(t) = 0 \Rightarrow$$

$$\sum_{k=1}^b$$

$$a_{jk} i_k(t) = 0$$

$j = \text{rows} = \text{nodes} = 1, \dots, (n+1)$

$k = \text{columns} = \text{branches} = 1, \dots, b$

A → Reduced Incidence Matrix

From $Ai(t) = 0$

Re-writing A in the Form $[A_t \ A_L]$

And Re-writing $i(t)$ in the Form $\begin{bmatrix} i_t \\ i_L \end{bmatrix}$

where A_t is the $n \times n$ Square Matrix Corresponding To Twigs

And A_L is the $n \times (b-n)$ Matrix Corresponding To Links

Hence; $Ai(t) = 0 \Rightarrow [A_t \ A_L] \begin{bmatrix} i_t \\ i_L \end{bmatrix} = 0$

for twigs
for links

Since We're Focusing On The Tree:

∴ KCL: $A_t i_t(t) = 0$

From The Tree, The Complete Incidence Matrix, A_a is:

Nodes
↓

$$A_a = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & -1 & 1 & 1 & 0 \\ 3 & 0 & 0 & 0 & 0 & -1 & 1 \\ 4 & -1 & -1 & 0 & -1 & 0 & -1 \end{bmatrix}$$

Eliminating The Ground Node i.e. Last Row To Get The Reduced Incidence Matrix, $[A]$.

Node
↓

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & -1 & 1 & 1 & 0 \\ 3 & 0 & 0 & 0 & -1 & 1 & 1 \end{bmatrix}$$

Re-writing A as $[A_t \ A_L]$

Node
↓

$$A = \begin{bmatrix} 2 & 4 & 6 & 1 & 3 & 5 & 7 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 \end{bmatrix}$$

Twigs Links

↓ ↓

A_t A_L

KCL: ↓

$Ai(t) = 0 \Leftrightarrow \begin{cases} i_1(t) + i_2(t) + i_3(t) = 0 \\ -i_3(t) + i_4(t) + i_5(t) = 0 \\ -i_5(t) + i_6(t) + i_7(t) = 0 \end{cases}$

Hence $A_t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ AND $A_L = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

Notice A_t is A UNIT MATRIX

∴ The KCL of The CAPACITOR TREE is: where Tree Branches Are {2, 4, 6}

$$A_t i_t(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_2(t) \\ i_4(t) \\ i_6(t) \end{bmatrix} = 0$$

• Now, To WRITE THE KVL EQUATION:

KVL is given AS $B V(t) = 0 \Rightarrow \sum_{k=1}^b b_{jk} V_k(t) = 0$

where $j = \text{rows} = \text{loops} = 1, 2, \dots, \text{all loops}$
 $k = \text{columns} = \text{branches} = 1, 2, \dots, n$

Note that B is the Tie-Set OR f-loop Matrix.

Rewriting B AS $[B_t \quad B_l]$

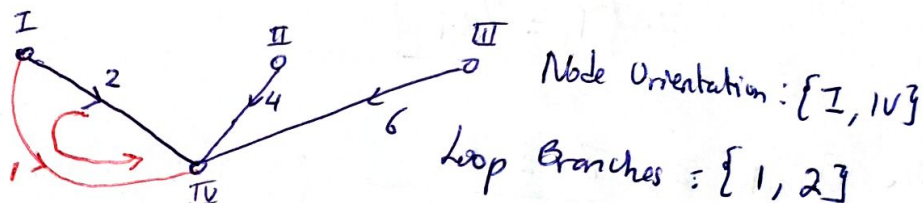
AND Re-writing $V(t)$ in the form $\begin{bmatrix} V_t \\ V_l \end{bmatrix}$

Since we're focusing on Tree, We use the Sub-matrices Corresponding To the Twigs.

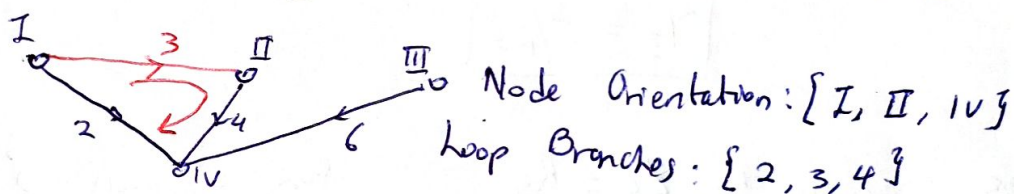
• For the Capacitor Tree; Twigs $\rightarrow \{2, 4, 6\}$
 Links $\rightarrow \{1, 3, 5, 7\}$

To Get the Loop Matrix:

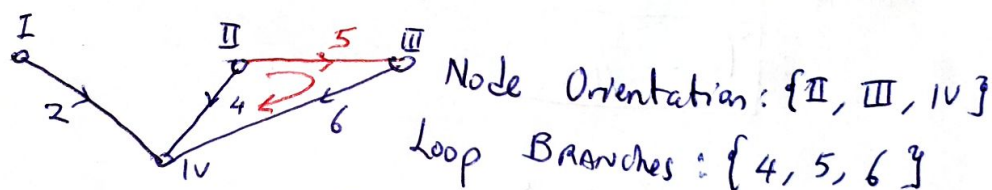
→ PUTTING LINK 1:



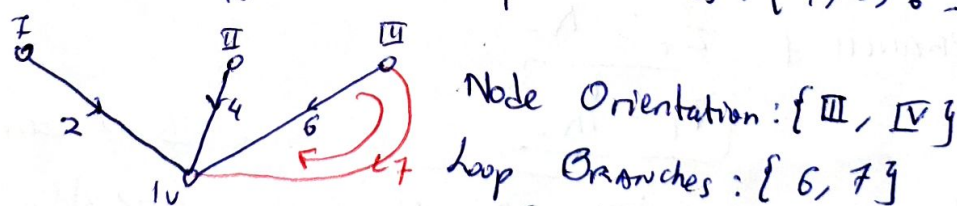
→ PUTTING LINK 3:



→ PUTTING LINK 5:



→ PUTTING LINK 7:



Hence The f-loop MATRIX:

(or Tie-Set MATRIX)

KVL: $B_f V = 0 \Rightarrow$

$B_f =$

f-loops	Branches	1	2	3	4	5	6	7
1		1	-1	0	0	0	0	0
2		0	-1	1	1	0	0	0
3		0	0	0	-1	1	1	0
4		0	0	0	0	0	-1	1

Re-writing B_f in the form:

$B_f = [B_t \quad B_l]$

$B_t =$

f-loops	Twigs	2	4	6
1		1	0	0
2		-1	1	0
3		0	-1	1
4		0	0	-1

$B_l =$

Links	1	3	5	7
1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1

Hence For the Tree KVL, Use $B_t = \begin{bmatrix} -1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$

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Hence $B_t V_t(t) \Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_2(t) \\ V_4(t) \\ V_6(t) \end{bmatrix}$ where The Tree Branches Are $\{2, 4, 6\}$

c.) To Give The Diagonal 7×7 Matrix of Branch By Branch Admittance Matrix:

from Ohms Law $\rightarrow Z = V/I$ where $Z \rightarrow$ Impedance
but $Y \rightarrow 1/Z$
Hence; $V = I \times Z$
Hence; $V = I \times 1/Y$
 $\therefore \boxed{I = YV}$

Recall for A Capacitor;

$$Z_c = 1/sC =$$

$$\text{where } 1/sC = 1/j\omega C$$

$$\therefore Z_c = 1/sC$$

$$\therefore Y = sC$$

For Branch 1; $Z_1 = R_1$; For Branch 2; $Z_2 = 1/sC_2$
 $\therefore \boxed{Y_1 = 1/R_1}$ $\therefore \boxed{Y_2 = sC_2}$

For Branch 3; $Z_3 = R_3$; For Branch 4; $Z_4 = 1/sC_4$
 $\therefore \boxed{Y_3 = 1/R_3}$ $\therefore \boxed{Y_4 = sC_4}$

For Branch 5; $Z_5 = R_5$; For Branch 6; $Z_6 = 1/sC_6$
 $\therefore \boxed{Y_5 = 1/R_5}$ $\therefore \boxed{Y_6 = sC_6}$

For Branch 7; $Z_7 = R_7$
 $\therefore \boxed{Y_7 = 1/R_7}$

Voltage Source vector



where $I \rightarrow$ Current Vector

$Y \rightarrow$ Admittance Matrix

$V \rightarrow$ Voltage Vector

Hence from $\boxed{I = Y_b(s)V}$

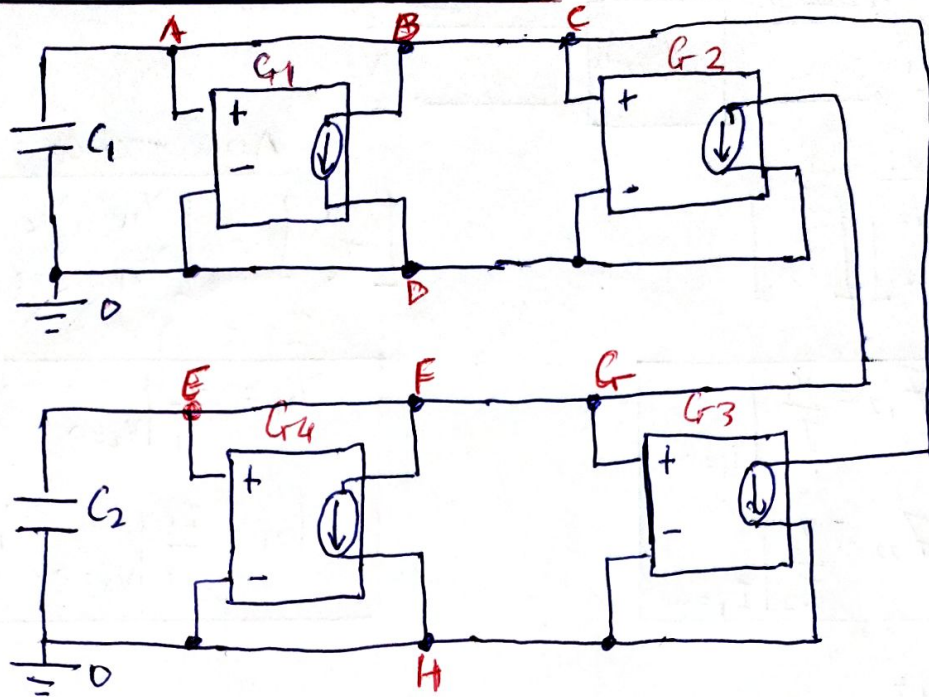
$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \end{bmatrix} = \begin{bmatrix} 1/R_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & sC_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/R_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & sC_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/R_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & sC_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1/R_7 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \end{bmatrix}$$

Observe The Diagonal Matrix $Y_b(s)$ in frequency DOMAIN
 $\rightarrow Y_b(s) \rightarrow$ Branch by Branch Admittance Matrix

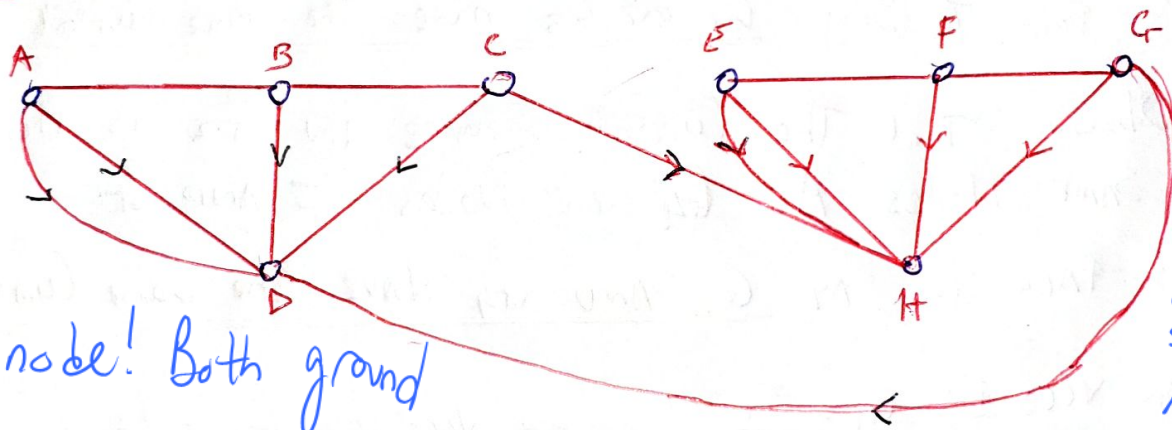
QUESTION 3

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The Circuit:



a.) The GRAPH OF THE CIRCUIT ORIENTING ALL BRANCHES TOWARD
GROUND IS:



D & H are same node! Both ground

← This is not correct, see next page

b.) To Get The Non-Diagonal 2×2 2-PORT ADMITTANCE MATRIX, (Y_G) , Seen To The Right Of The Capacitors. ^{This (b) is not correct too}
→ Using The VOLTAGE CONTROLLED CURRENT SOURCES (VCCS) GAINS AS:
 $G_i = 1, 2, 3, 4$

Recall That $G_{\text{gain}} = \frac{\text{Output}}{\text{Input}}$

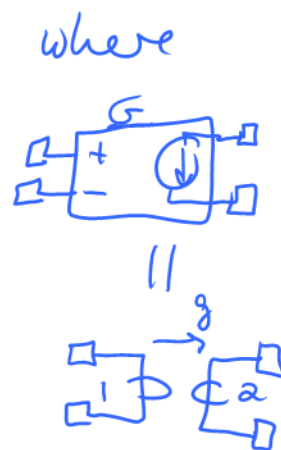
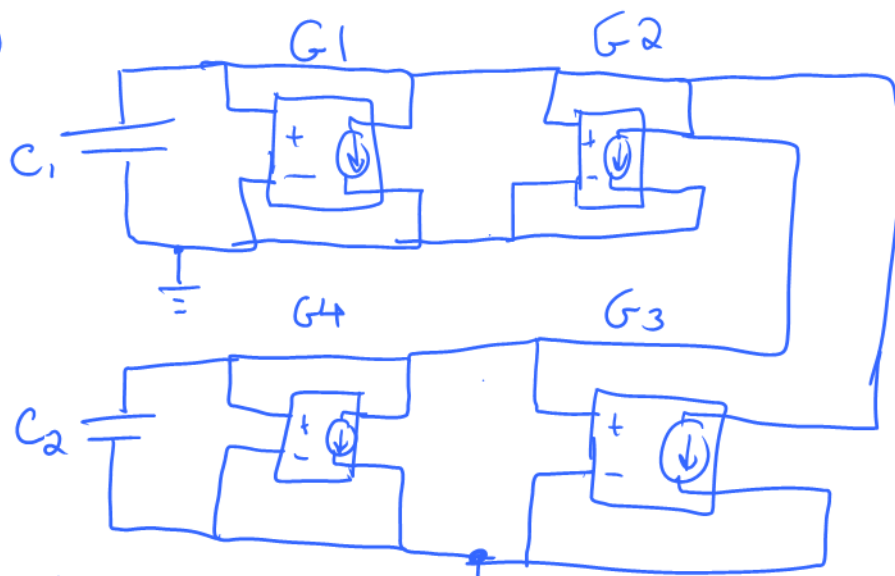
And for A VCCS, $\text{output} = \text{Current}$
 $\text{input} = \text{Voltage}$

Hence; For A VCCS, $G_{\text{AIN}} = \frac{\text{Output}}{\text{Input}} = \frac{I}{V} = \frac{\text{Current}}{\text{Voltage}}$

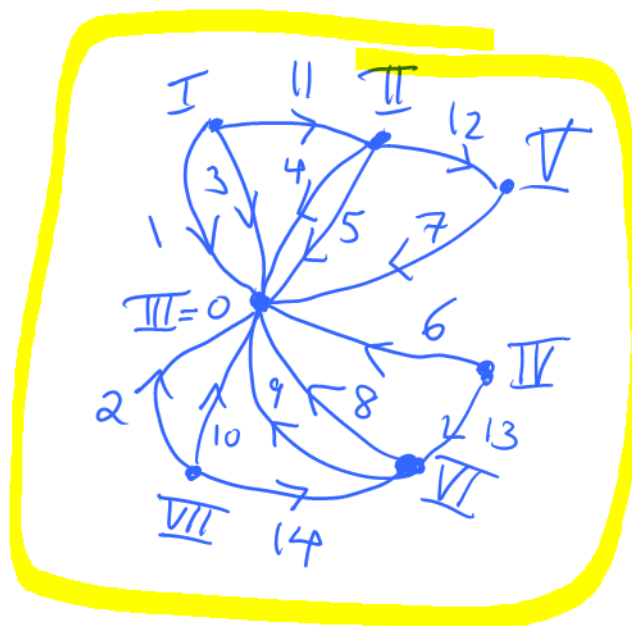
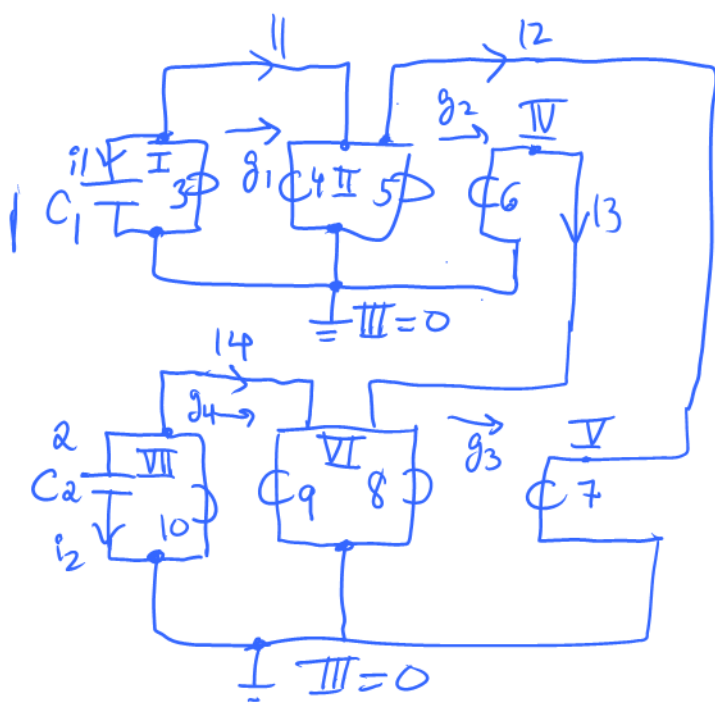
Hence G_i Can Be Seen To Be An Admittance.

Since for Admittance $Y = \frac{I}{V}$

③ a)



Equivalent Circuits:



b) Given $Y_{Gi} = \begin{bmatrix} 0 & -g_i \\ g_i & 0 \end{bmatrix}$ for $i=1,2,3,4$ & $Y_{Cj} = \begin{bmatrix} C_j s & -C_j s \\ -C_j s & C_j s \end{bmatrix}$ $j=1,2$

Get $Y_C(s) + Y_G$ seen looking into the capacitor

nodes from the outside. Give the degree 2 polynomial in $s = \det(Y_G)$
 its roots are (open circuit) natural frequencies due to initial conditions.
 Calculate natural frequencies when all C 's have value 1 (1F) and
 all G 's have value -1 (-1 Ω)

$$Y_G = Y_{G1} \parallel Y_{G2} \parallel Y_{G3} \parallel Y_{G4} = Y_{G1} + Y_{G2} + Y_{G3} + Y_{G4} = \begin{bmatrix} 0 & -(g_1 + g_2 + g_3 + g_4) \\ g_1 + g_2 + g_3 + g_4 & 0 \end{bmatrix}$$

$$Y_C(s) = Y_{C1} + Y_{C2} = \begin{bmatrix} C_1 s & -C_1 s \\ -C_1 s & C_1 s \end{bmatrix} + \begin{bmatrix} C_2 s & -C_2 s \\ -C_2 s & C_2 s \end{bmatrix} = \begin{bmatrix} (C_1 + C_2) s & -(C_1 + C_2) s \\ -(C_1 + C_2) s & (C_1 + C_2) s \end{bmatrix}$$

$$\Rightarrow Y(s) = Y_o + Y_c(s) = \begin{bmatrix} (c_1+c_2)s & -(g_1+g_2+g_3+g_4+(c_1+c_2)s) \\ g_1+g_2+g_3+g_4-(c_1+c_2)s & (c_1+c_2)s \end{bmatrix}$$

$$|Y(s)| = \begin{vmatrix} (c_1+c_2)s & -(g_1+g_2+g_3+g_4+(c_1+c_2)s) \\ g_1+g_2+g_3+g_4-(c_1+c_2)s & (c_1+c_2)s \end{vmatrix}$$

$$= (c_1+c_2)^2 s^2 - [g_1+g_2+g_3+g_4+(c_1+c_2)s][g_1+g_2+g_3+g_4-(c_1+c_2)s]$$

$$= (c_1+c_2)^2 s^2 - (g_1+g_2+g_3+g_4)^2 + (c_1+c_2)^2 s^2$$

$$|Y(s)| = (g_1+g_2+g_3+g_4)^2$$

When $g_1=g_2=g_3=g_4=-1$: $\det(Y(s)) = 16$

ALSO RECALL THAT FOR A 2-port Network:



IMPEDANCE MATRIX:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V = ZI$$

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0}, \quad Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0}$$

$$Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0}, \quad Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$

ADMITTANCE MATRIX

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I = YV$$

$$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}, \quad Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0}$$

$$Y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0}, \quad Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0}$$

From the CIRCUIT:

→ Observe the CURRENT SOURCE IN G_3 PICKS FROM THE SAME NODES AS G_1 i.e. Nodes B and C

Hence, IT'S FAIR TO SAY G_1 and G_3 have the Same Current I_1

→ Also, Observe That The Current Source IN G_2 IS PICKING FROM THE SAME NODES AS G_4 i.e. Nodes F AND G

Hence, IT'S FAIR TO SAY G_2 AND G_4 HAVE THE SAME CURRENT I_2

Hence, For VCCS 1; $G_1 = \frac{I_1}{V_1}$

For VCCS 2; $G_2 = \frac{I_2}{V_1}$

$$\therefore I_1 = G_1 V_1$$

$$I_2 = G_2 V_1$$

Note That G_1 IS PARALLEL WITH G_2 SO THEY HAVE THE SAME VOLTAGE V_1
Also, G_3 IS PARALLEL WITH G_4 SO THEY HAVE THE SAME VOLTAGE V_2

For VCCS 3; $G_3 = \frac{I_1}{V_2}$

For VCCS 4; $G_4 = \frac{I_2}{V_2}$

$$\therefore I_1 = G_3 V_2$$

$$I_2 = G_4 V_2$$

$$\therefore \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} G_1 & G_3 \\ G_2 & G_4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\therefore Y_G = \begin{bmatrix} G_1 & G_3 \\ G_2 & G_4 \end{bmatrix}$$

Now To determine The Diagonal Admittance of The Capacitors ;

Considering The Capacitors Branch by Branch :

For CAPACITOR C_1 whereas For Capacitor C_2

$$Z_1 = 1/sC_1$$

$$Z_2 = 1/sC_2$$

$$\therefore Y_1 = 1/Z_1 = sC_1$$

$$Y_2 = 1/Z_2$$

$$I_2 = Y_2 V_2$$

$$\therefore I_1 = Y_1 V_1$$

$$Y_2 = sC_2$$

So From $I = Y V$

$$\therefore \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} sC_1 & 0 \\ 0 & sC_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\therefore Y_C(s) = \begin{bmatrix} sC_1 & 0 \\ 0 & sC_2 \end{bmatrix}$$

The Total 2-Port Admittance, $Y(s) = Y_C(s) + Y_G$

$$\therefore Y(s) = \begin{bmatrix} sC_1 & 0 \\ 0 & sC_2 \end{bmatrix} + \begin{bmatrix} G_1 & G_3 \\ G_2 & G_4 \end{bmatrix}$$

$$Y(s) = \begin{bmatrix} sC_1 + G_1 & G_3 \\ G_2 & sC_2 + G_4 \end{bmatrix}$$

$$\text{Hence; } \det[Y(s)] = \begin{vmatrix} sC_1 + G_1 & G_3 \\ G_2 & sC_2 + G_4 \end{vmatrix}$$

$$= (sC_1 + G_1)(sC_2 + G_4) - G_2 G_3$$

$$\therefore \det(Y(s)) = s^2 C_1 C_2 + s(C_1 G_4 + C_2 G_1) + G_1 G_4 - G_2 G_3$$

2nd degree
Polynomial

$$Y(s) = s^2 C_1 C_2 + s(C_1 G_4 + C_2 G_1) + (G_1 G_4 - G_2 G_3)$$

The roots i.e "S" of the Quadratic Equation OR 2nd degree polynomial ARE NATURAL FREQUENCIES DUE TO INITIAL CONDITIONS

Using Quadratic Formula

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$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{FOR } ax^2 + bx + c = 0$$

Comparing with $s^2 C_1 C_2 + s(C_1 G_4 + G_2 G_1) + (G_1 G_4 - G_2 G_3)$

$$\therefore a = C_1 C_2$$

$$b = C_1 G_4 + G_2 G_1$$

$$c = G_1 G_4 - G_2 G_3$$

By Substitution

$$S = \frac{-(C_1 G_4 + G_2 G_1) \pm \sqrt{(C_1 G_4 + G_2 G_1)^2 - 4(C_1 C_2)(G_1 G_4 - G_2 G_3)}}{2 C_1 C_2}$$

where Root of S are Natural Frequencies Due to Initial Conditions.

If All Values of $C = 1F$

And All Values of $G = 1 \text{ mho}$ OR Ω^{-1}

• mho is the reciprocal of ohm

$$\therefore a = C_1 C_2 = 1 \times 1$$

$$\boxed{a = 1}$$

Also $b = C_1 G_4 + G_2 G_1 = 1 \times (-1) + (1 \times -1)$

$$\boxed{b = -2}$$

And $c = G_1 G_4 - G_2 G_3$

$$c = (-1)(-1) - (-1)(-1) = 1 - 1$$

$$\boxed{c = 0}$$

Hence; $S = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(0)}}{2(1)}$

$$S = \frac{2 \pm \sqrt{4}}{2}$$

$$S = \frac{2 \pm 2}{2}$$

$$S = \frac{2+2}{2} \quad \text{OR} \quad \frac{2-2}{2}$$

$$S = 2 \quad \text{OR} \quad 0$$

$$\therefore \boxed{S = 2}$$

C.) Draw A degree 3 Generalization
Having 3 Capacitors And 9 Voltage Controlled Current Source

