

"Homework 10"

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Problem 1)

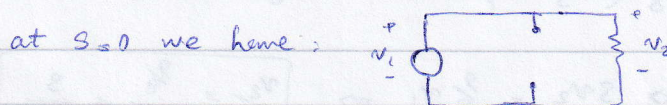
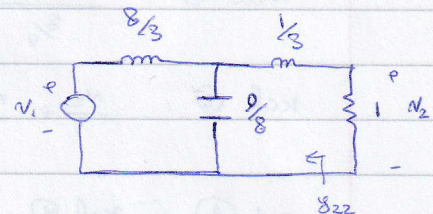
$$a) T(s) = \frac{K_a}{s^3 + 3s^2 + 3s + 1} = \frac{\frac{K_a}{s^3 + 3s}}{1 + \frac{3s^2 + 1}{s^3 + 3s}}$$

→ it has 3 zeroes at ∞

$$\Rightarrow y_{22} = \frac{3s^2 + 1}{s^3 + 3s}$$

$$\begin{array}{r} \frac{1}{3}s \\ \hline 3s^2 + 1 \quad \frac{s^3 + 3s}{s^3 + \frac{1}{3}s} \\ \hline \frac{8}{3}s \quad \frac{9}{8}s \\ \hline \frac{8}{3}s \quad \frac{3s^2 + 1}{3s^2} \\ \hline \quad \frac{8}{3}s \\ \hline \quad \frac{8}{3}s \\ \hline \quad 0 \end{array}$$

$$\Rightarrow y_{22} = \frac{1}{\frac{1}{3}s + \frac{1}{\frac{9}{8}s + \frac{1}{\frac{8}{3}s}}}$$



$$\text{at } s=0 \Rightarrow V_1 = V_2$$

$$\Rightarrow T(s) \Big|_{s=0} = 1 \Rightarrow \boxed{K_a = 1}$$

$$b) T(s) = \frac{K_b s^2}{s^3 + 3s^2 + 3s + 1} = \frac{\frac{K_b s^2}{s^3 + 3s}}{1 + \frac{3s^2 + 1}{s^3 + 3s}}$$

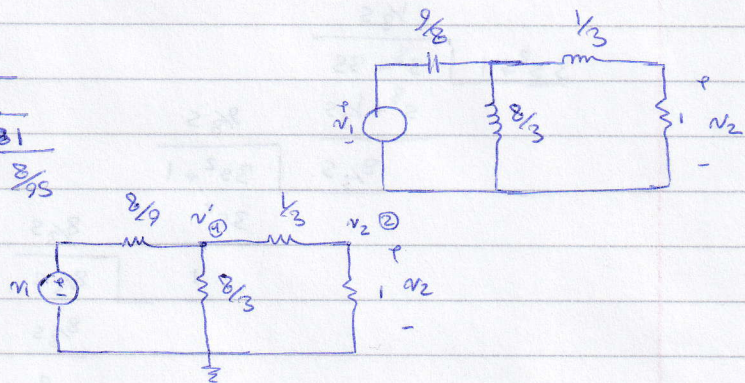
→ it has 2 zeroes at $s=0$ & 1 zero at ∞

$$\Rightarrow y_{22} = \frac{3s^2 + 1}{s^3 + 3s}$$

$$\begin{array}{r}
 \frac{1}{3}s \\
 3s^2+1 \overline{) s^3+3s} \\
 \underline{s^3+\frac{1}{3}s} \\
 \frac{8}{3}s \\
 \underline{1+3s^2} \\
 1 \\
 3s^2 \overline{) \frac{8}{3}s} \\
 \underline{\frac{8}{3}s} \\
 0
 \end{array}$$

$$y_{22} = \frac{1}{\frac{1}{3}s + \frac{1}{\frac{3}{8}s + \frac{81}{8/s}}}$$

at $s=1$ we will have:



$$\text{Kcl } ①: \frac{v'-v_1}{8/9} + \frac{v'}{8/3} + \frac{v'-v_2}{1/3} = 0 \Rightarrow \frac{36}{8}v' - 3v_2 = \frac{9}{8}v_1$$

$$\text{Kcl } ②: \frac{v_2}{1} + \frac{v_2-v'}{1/3} = 0 \Rightarrow v_2 + 3(v_2-v') = 0 \Rightarrow 4v_2 = 3v' \Rightarrow \boxed{v' = \frac{4}{3}v_2} \quad (A)$$

$$\text{put } (A) \text{ in Kcl } ①: \frac{36}{8} \left(\frac{4}{3}v_2 \right) - 3v_2 = \frac{9}{8}v_1 \Rightarrow 6v_2 - 3v_2 = \frac{9}{8}v_1$$

$$\Rightarrow 3v_2 = \frac{9}{8}v_1 \Rightarrow \boxed{\frac{v_2}{v_1} = \frac{9/8}{3} = \frac{3}{8}}$$

$$T(s) \big|_{s=1} = \frac{3}{8} \Rightarrow \frac{k_b}{8} = \frac{3}{8} \Rightarrow \boxed{k_b = 3}$$

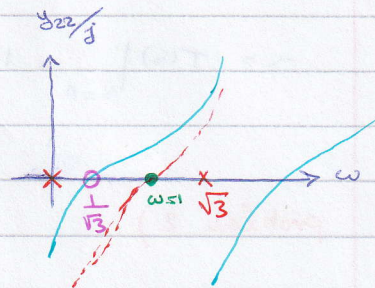
$$c) T(s) = \frac{k_c (s^2+1)}{s^3+3s^2+3s+1}$$

$\Rightarrow$ it has 2 zeroes on (jw) axis & 1 zero at ∞

$$T(s) = \frac{K_C (s^2 + 1)}{s^3 + 3s}$$

$$y_{22} = \frac{3s^2 + 1}{s^3 + 3s} \rightarrow \text{Zero at } \omega = \frac{1}{\sqrt{3}}$$

$$\rightarrow \text{poles at } \omega = 0, \sqrt{3}$$



$$y_{22} = \frac{k_1}{s} + \frac{2k_2 s}{s^2 + 3}$$

$$k_1 = s y_{22} \Big|_{s=0} = \frac{3s^2 + 1}{s^2 + 3} \Big|_{s=0} = \frac{1}{3}$$

$$2k_2 = \frac{s^2 + 3}{s} y_{22} \Big|_{s^2 = -3} = \frac{3s^2 + 1}{s^2} \Big|_{s^2 = -3} = \frac{8}{3}$$

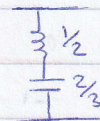
$$\Rightarrow y_{22} = \frac{1/3}{s} + \frac{8/3 s}{s^2 + 3}$$

partial pole removal: $y_{22} - \frac{\hat{k}s}{s^2 + 3} \Big|_{s^2 = -1} = \left(\frac{1}{3s} + \frac{8/3 s}{s^2 + 3} - \frac{\hat{k}s}{s^2 + 3} \right) \Big|_{s^2 = -1} = 0$

$$\frac{s}{s^2 + 3} \left(\frac{1}{3} \frac{s^2 + 3}{s^2} + \frac{8/3 - \hat{k}}{s^2 + 3} \right) \Big|_{s^2 = -1} = 0$$

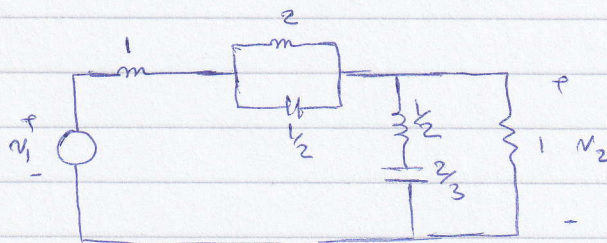
$$\Rightarrow \boxed{\hat{k} = 8/3 - 2/3 = 2}$$

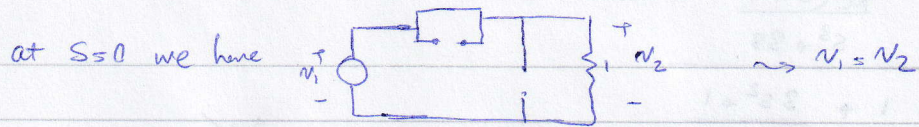
$$y_1 = \frac{2s}{s^2 + 3} = \frac{1}{\frac{s}{2} + \frac{3}{2s}}$$



$$\text{error } y_2 = y_{22} - y_1 = \frac{s^2 + 1}{s(s^2 + 3)}$$

$$\rightarrow z_2 = \frac{s(s^2 + 3)}{s^2 + 1} = s + \frac{2s}{s^2 + 1} = s + \frac{1}{s/2 + 1/2s}$$





$$\rightarrow T(s) \Big|_{s=0} = 1 \rightarrow \frac{K_C}{1} = 1 \rightarrow \boxed{K_C = 1}$$

problem 2)

a)

$$\underbrace{\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}}_C \frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \underbrace{\begin{bmatrix} -3 & 2 & 3 \\ 0 & -2 & 0 \\ -3 & 0 & -1 \end{bmatrix}}_A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 2 & 0 \\ 0 & 3 \\ 3 & 2 \end{bmatrix}}_B \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$v_0 = \underbrace{\begin{bmatrix} 1 & 1 & 2 \end{bmatrix}}_C \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 4 \end{bmatrix}}_D \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$Y_C = \left[\begin{array}{ccc|cc} +3 & -2 & -3 & -2 & 0 & 0 \\ 0 & 2 & 0 & 0 & -3 & 0 \\ 3 & 0 & 1 & -3 & -2 & 0 \\ \hline 2 & 0 & 3 & 0 & 0 & 0 \\ 0 & 3 & 2 & 0 & 0 & 4 \\ \hline -1 & -1 & -2 & 0 & -4 & 1 \end{array} \right]$$

$$T(s) = [1 \ 1 \ 2] \left(\begin{bmatrix} 2s & 0 & 0 \\ 0 & 4s & 0 \\ 0 & 0 & 6s \end{bmatrix} - \begin{bmatrix} -3 & 2 & 3 \\ 0 & -2 & 0 \\ -3 & 0 & - \end{bmatrix} \right)^{-1} \begin{bmatrix} 2 & 0 \\ 0 & 3 \\ 3 & 2 \end{bmatrix} + [0 \ 4]$$

$$= [1 \ 1 \ 2] \begin{bmatrix} 2s+3 & -2 & -3 \\ 0 & 4s+2 & 0 \\ 3 & 0 & 6s+1 \end{bmatrix}^{-1} \begin{bmatrix} 2 & 0 \\ 0 & 3 \\ 3 & 2 \end{bmatrix} + [0 \ 4]$$

$$= [1 \ 1 \ 2] \cdot \begin{bmatrix} \frac{6s+1}{4(3s^2+5s+3)} & \frac{6s+1}{4(6s^3+13s^2+11s+3)} & \frac{3}{4(3s^2+5s+3)} \\ 0 & \frac{1}{2(2s+1)} & 0 \\ \frac{-3}{4(3s^2+5s+3)} & \frac{-3}{4(6s^3+13s^2+11s+3)} & \frac{2s+3}{4(3s^2+5s+3)} \end{bmatrix}$$

$$\cdot \begin{bmatrix} 2 & 0 \\ 0 & 3 \\ 3 & 2 \end{bmatrix} + [0 \ 4]$$

$$= \left[\frac{9(2s+3)}{4(3s^2+5s+3)} + \frac{6s+1}{2(3s^2+5s+3)} - \frac{9}{4(3s^2+5s+3)} \right],$$

$$\frac{3(2s+3)}{2(3s^2+5s+3)} + \frac{3(6s+1)}{4(6s^3+13s^2+11s+3)} + \frac{3}{2(2s+1)} + \frac{3}{2(3s^2+5s+3)}$$

$$- \frac{27}{4(6s^3+13s^2+11s+3)} + 4]$$

$$= \left[\frac{30s+20}{4(3s^2+5s+3)}, \frac{6s+12}{2(3s^2+5s+3)} + \frac{18s-24}{4(6s^3+13s^2+11s+3)} + \frac{3}{2(2s+1)} + 4 \right]$$