Homework 6

Estefany Carrillo
ENEE 610

1. Synthesize $y(s) = \frac{3}{2} \cdot \frac{s(s^2+9)}{(s^2+4)}$ at $k=1,3$

Let $k=1$,

$$R(s) = \frac{k\,z(s) - s\,z(k)}{k\,z(k) - s\,z(s)}$$

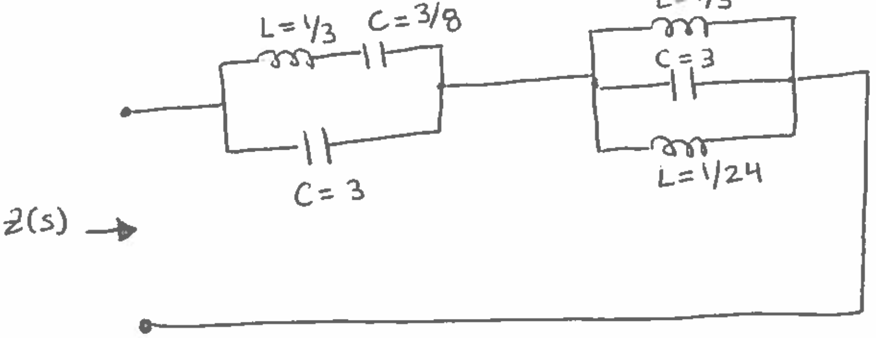
$$Z(s) = \frac{1}{\frac{1}{Z(k)R(s)} + \frac{s}{kZ(k)}} + \frac{1}{\frac{k}{sZ(k)} + \frac{R(s)}{Z(k)}}$$

$$Z(s) = \frac{1}{y(s)} = \frac{2}{3} \cdot \frac{(s^2+4)}{s(s^2+9)} \Rightarrow Z(1) = \frac{2}{3} \cdot \frac{5}{10} = \frac{1}{3}$$

$$R(s) = \frac{\frac{2}{3} \cdot \frac{(s^2+4)}{s(s^2+9)} - s \cdot \frac{1}{3}}{\frac{1}{3} - \frac{2}{3} \cdot \frac{(s^2+4)}{(s^2+9)}} = \frac{\frac{2(s^2+4) - s^2(s^2+9)}{3s(s^2+9)}}{\frac{s^2+9-2s^2-8}{3(s^2+9)}}$$

$$= \frac{s^4+7s^2-8}{s^3-s} = \frac{(s^2+8)(s^2-1)}{s(s+1)(s-1)} = s + \frac{8}{s}$$

$$Z(s) = \frac{1}{\frac{1}{\frac{s}{3} + \frac{8}{3s}} + \frac{s}{\frac{1}{3}}} + \frac{1}{\frac{1}{\frac{s}{3}} + 3s + \frac{24}{s}}$$



Estefany Carrillo
ENEE610

Let $k=3$,

$$Z(3) = \frac{\overset{1}{\cancel{2}}}{3}\cdot\frac{13}{3(\cancel{18}_{9})} = \frac{13}{27}\cdot\frac{1}{3} = \frac{13}{81}$$

$$R(s) = \frac{3\cdot\frac{2}{3}\cdot\frac{(s^2+4)}{s(s^2+9)} - \frac{13}{81}s}{3\cdot\frac{13}{81} - s\cdot\frac{2}{3}\frac{(s^2+4)}{s(s^2+9)}}$$

$$= \frac{\frac{162(s^2+4)-13s^2(s^2+9)}{81s(s^2+9)}}{\frac{13(s^2+9)-18(s^2+4)}{27(s^2+9)}} = \frac{1}{3}\cdot\frac{45s^2+648-13s^4}{s(-5s^2+45)}$$

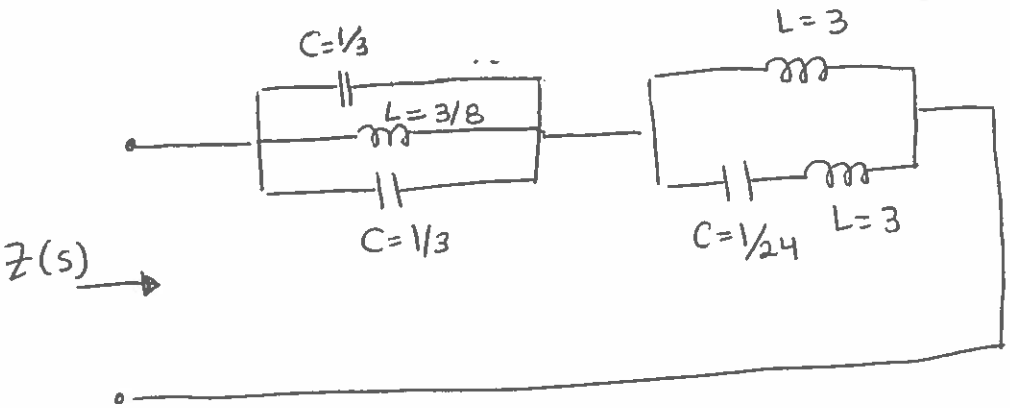
$$= \frac{1}{3}\cdot\frac{13s^4-45s^2-648}{5s(s^2-9)} = \frac{1}{3}\cdot\frac{(13s^2+72)(s-3)(s+3)}{5s(s-3)(s+3)}$$

$$= \frac{1}{3}\cdot\frac{13}{5}s + \frac{24}{5}\cdot\frac{1}{s} = \frac{13}{15}s + \frac{24}{5s}$$

$$Z(s) = \frac{1}{\frac{1}{\frac{169}{1215}s+\frac{104}{135s}} + \frac{s}{\frac{\cancel{39}\,13}{\cancel{81}\,27}}} + \frac{1}{\frac{3}{\frac{13}{81}s} + \frac{\frac{13}{15}s+\frac{24}{5s}}{\frac{13}{81}}}$$

$$= \frac{1}{\frac{1}{\frac{169}{1215}s+\frac{104}{135s}} + \frac{s}{\frac{13}{27}}} + \frac{1}{\frac{243}{13s} + \frac{27}{5}s + \frac{1944}{65s}}$$

$$= \frac{1}{\frac{s}{3}+\frac{8}{3s}+\frac{s}{3}} + \frac{1}{\frac{1}{3s}+\frac{1}{3s+\frac{24}{s}}}$$

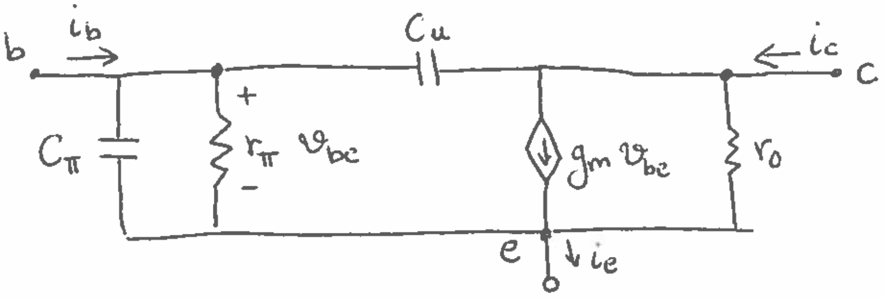


In comparison with results for y(s), the C's are replaced with L's and vice-versa, but they have different values.

2.

$$Y(s) := \begin{bmatrix} g\pi + s.(C\pi + Cu) & -sCu \\ -sCu + gm & g_o + sCu \end{bmatrix}$$

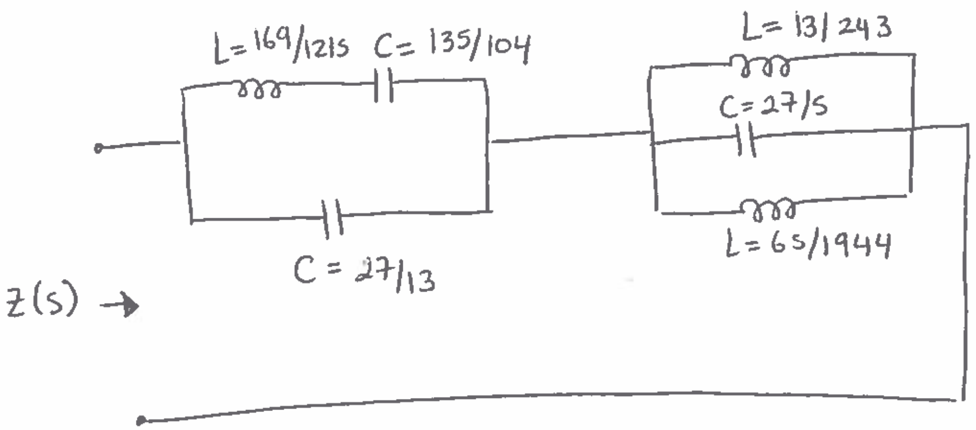
a)



$$i_b = \frac{v_{be}}{r_\pi} + sC_\pi v_{be} + sC_u v_{bc}$$

$$i_c = g_m v_{be} + \frac{v_{ce}}{r_o} - sC_u v_{bc}$$

$$i_e = -\frac{v_{be}}{r_\pi} - sC_\pi v_{be} - g_m v_{be} - \frac{v_{ce}}{r_o}$$



→ Results for k=1 and k=3 are different, same circuit but with different values for L's and C's.

b) $z(s) = \left(\frac{3}{2}\right) \frac{s(s^2+9)}{(s^2+4)}$

Let k=1, $z(1) = \frac{3}{2} \cdot \frac{10}{5} = 3$

$$R(s) = \frac{k\,z(s) - s\,z(k)}{k\,z(k) - s\,z(s)} = \frac{\frac{3}{2}\,\frac{s(s^2+9)}{(s^2+4)} - 3s}{3 - \frac{3}{2}\,\frac{s^2(s^2+9)}{(s^2+4)}}$$

$$= \frac{\frac{3s^3+27s-6s^3-24s}{2(s^2+4)}}{\frac{6s^2+24-3s^4-27s^2}{2(s^2+4)}} = \frac{3s^3-3s}{21s^2+3s^4-24}$$

$$= \frac{3s(s^2-1)}{3(s^4+7s^2-8)} = \frac{s}{s^2+8} = \frac{1}{s+\frac{8}{s}}$$

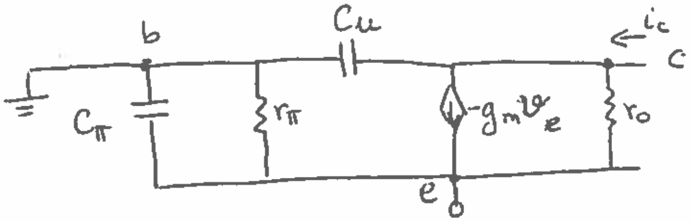
$$z(s) = \frac{1}{\frac{1}{z(k)R(s)} + \frac{s}{kz(k)}} + \frac{1}{\frac{k}{s\,z(k)} + \frac{R(s)}{z(k)}}$$

$$= \frac{1}{\frac{1}{\frac{3}{s+\frac{8}{s}}} + \frac{s}{3}} + \frac{1}{\frac{1}{3s} + \frac{1}{3s+\frac{24}{s}}}$$

$g_\pi = \frac{1}{r_\pi}$, $g_o = \frac{1}{r_o}$

$$\begin{bmatrix} i_b \\ i_c \\ i_e \end{bmatrix} = \underbrace{\begin{bmatrix} g_\pi + sC_\pi & sC_\mu & 0 \\ g_m & -sC_\mu & g_o \\ -g_\pi - sC_\pi - g_m & 0 & -g_o \end{bmatrix}}_{Y_{ind}} \begin{bmatrix} v_{be} \\ v_{bc} \\ v_{ce} \end{bmatrix}$$

b)



$$i_b = -\frac{v_e}{r_\pi} - sC_\pi v_e - sC_\mu v_c$$

$$i_c = -g_m v_e + \frac{v_{ce}}{r_o} + sC_\mu v_c$$

$$i_e = \frac{v_e}{r_\pi} + sC_\pi v_e + g_m v_e - \frac{v_{ce}}{r_o}$$

$$\begin{bmatrix} i_e \\ i_c \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} g_\pi + sC_\pi + g_m + g_o & -g_o \\ -g_m - g_o & g_o + sC_\mu \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

c)

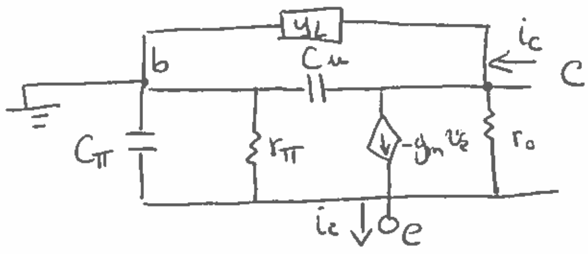
$$\begin{bmatrix} i_e \\ i_c \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ -y_L v_2 \end{bmatrix} \rightsquigarrow$$

$y_{11} = g_\pi + sC_\pi + g_m + g_o$

$y_{12} = -g_o$

$y_{21} = -g_m - g_o$

$y_{22} = g_o + sC_\mu$



$$y_{11} v_1 + y_{12} v_2 = i_1$$

$$y_{21} v_1 + y_{22} v_2 = -y_L v_2$$

$$y_{21} v_1 = -(y_L + y_{22}) v_2$$

$$\Rightarrow v_2 = -\frac{y_{21}}{y_L + y_{22}}$$

$$y_{11} v_1 - \frac{y_{12} y_{21}}{y_L + y_{22}} v_1 = i_1$$

$$y_{in} = \frac{i_1}{v_1} = y_{11} - \frac{y_{21} y_{12}}{y_L + y_{22}}$$

$$y_{in} = g_\pi + sC_\pi + g_m + g_o - \frac{(-g_m - g_o)(-g_o)}{y_L + (g_o + sC_\mu)}$$

$$= \big(g_\pi y_L + sC_\pi y_L + g_m y_L + g_o y_L + g_\pi g_o + sC_\pi g_o + \cancel{g_m g_o} + \cancel{g_o^2}$$

$$+ s g_\pi C_\mu + s^2 C_\pi C_\mu + s g_m C_\mu + s g_o C_\mu - \cancel{g_m g_o} - \cancel{g_o^2}\big) /$$

$$(y_L + g_o + sC_\mu)$$

$$= \Big[g_\pi (y_L + sC_\mu + g_o) + g_m (y_L + sC_\mu) + g_o (y_L + g_\pi + sC_\pi$$

$$+ sC_\mu) + sC_\pi y_L + s^2 C_\pi C_\mu\Big] \Big/ (y_L + g_o + sC_\mu)$$

3.

$$y(s) = \frac{s^2 + \frac{1}{2}s + \frac{1}{4}}{s^2 + \frac{1}{2}s + 1}$$

$$Z(s) = \frac{s^2 + \frac{1}{2}s + 1}{s^2 + \frac{1}{2}s + \frac{1}{4}}$$

Desire $\min_{\omega \geq 0} \operatorname{Re} y(j\omega) = G$

$$\operatorname{Re} y(j\omega) = [y(j\omega) + y(-j\omega)]/2$$

$$y(j\omega) = \frac{(-\omega^2 + \frac{1}{2}j\omega + 1/4)}{(-\omega^2 + \frac{1}{2}j\omega + 1)} \times \frac{(-\omega^2 - \frac{1}{2}j\omega + 1)}{(-\omega^2 - \frac{1}{2}j\omega + 1)}$$

$$\operatorname{Re} y(j\omega) = \frac{(-\omega^2 + 1/4)(-\omega^2+1) + \frac{1}{4}\omega^2}{(-\omega^2+1)^2 + \frac{1}{4}\omega^2} = \frac{\omega^4 - \frac{5}{4}\omega^2 + \frac{1}{4} + \frac{1}{4}\omega^2}{\omega^4 - 2\omega^2 + 1 + \frac{1}{4}\omega^2}$$

$$= \frac{\omega^4 - \omega^2 + 1/4}{\omega^4 - \frac{7}{4}\omega^2 + 1}$$

$$\left.\frac{d \operatorname{Re} y(j\omega)}{d\omega}\right|_{\omega_0} = 0 \quad , \quad \frac{d \operatorname{Re} y(j\omega)}{d\omega} = \left. - \frac{6\omega(4\omega^4 - 8\omega^2 + 3)}{(4\omega^4 - 7\omega^2 + 4)^2}\right|_{\omega_0} = 0$$

$$= \frac{-6\omega(2\omega^2-1)(2\omega^2-3)}{(4\omega^4 - 7\omega^2 + 4)^2}$$

$$\Rightarrow \omega_0 = \pm \frac{1}{\sqrt{2}}$$

$$\operatorname{Re} y(j\omega_0) = \frac{\frac{1}{4} - \frac{1}{2} + \frac{1}{4}}{\frac{1}{4} - \frac{7}{8} + 1} = 0 \Rightarrow G = 0 \Rightarrow y(s) - G = y(s)$$

$@\omega_0 = \frac{1}{\sqrt{2}}$

$$y(j\omega_0) = y(j\tfrac{1}{\sqrt{2}}) = \frac{-\frac{1}{2} + \frac{1}{4} + j\frac{1}{2\sqrt{2}}}{-\frac{1}{2} + 1 + j\frac{1}{2\sqrt{2}}} = \frac{\cancel{\tfrac{1}{2}}(-\frac{1}{2} + j\frac{1}{\sqrt{2}})}{\cancel{\tfrac{1}{2}}(1 + j\frac{1}{\sqrt{2}})} \times \frac{1 - j\frac{1}{\sqrt{2}}}{1 - j\frac{1}{\sqrt{2}}}$$

$$= \frac{(-\frac{1}{2} + \frac{1}{2}) + j\frac{1}{2\sqrt{2}} + j\frac{1}{\sqrt{2}}}{1 + (1/\sqrt{2})^2} = \frac{2}{3} \times \frac{1}{2\sqrt{2}}(2+1)j = (1/\sqrt{2})j$$

∴ Desire to cancel this term with the Richard's function

$$\left.\operatorname{Re} y(s)\right|_{s=j\omega_0} = 0 \quad , \quad \left.\operatorname{Im} y(s)\right|_{s=j\omega_0} = \frac{1}{\sqrt{2}}$$

$$y_R(s) = y(k)\left[\frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)}\right]$$

$$= y(k)\left[\frac{k\left[\frac{k^2+\frac{1}{2}k+\frac{1}{4}}{k^2+\frac{1}{2}k+1}\right] - s\left[\frac{s^2+\frac{1}{2}s+\frac{1}{4}}{s^2+\frac{1}{2}s+1}\right]}{k\left[\frac{s^2+\frac{1}{2}s+\frac{1}{4}}{s^2+\frac{1}{2}s+1}\right] - s\left[\frac{k^2+\frac{1}{2}k+\frac{1}{4}}{k^2+\frac{1}{2}k+1}\right]}\right]\Bigg|_{s=j\omega_0=j\frac{1}{\sqrt{2}}}$$

$$= y(k)\left[\frac{k\left(k^2+(1/2)k+1/4\right) - j\frac{1}{\sqrt{2}}\cdot j\frac{1}{\sqrt{2}}\left(k^2+1/2\,k+1\right)}{k\,j\frac{1}{\sqrt{2}}\left(k^2+\frac{1}{2}k+1\right) - j\frac{1}{\sqrt{2}}\left(k^2+\frac{1}{2}k+\frac{1}{4}\right)}\right]$$

$$y(s) = \frac{s^2+\frac{1}{2}s+\frac{1}{4}}{s^2+\frac{1}{2}s+1}\Bigg|_{s=j\frac{1}{\sqrt{2}}} = j\frac{1}{\sqrt{2}}$$

$$= y(k)\left[\frac{k^3+k^2+1/2\cdot k+1/2}{j\frac{1}{\sqrt{2}}\left\{k^3+\frac{1}{2}k^2+k-k^2-\frac{1}{2}k-\frac{1}{4}\right\}}\right]$$

$\Rightarrow$ Desire the denominator of this $= 0$

$$\Rightarrow\quad k^3-\frac{1}{2}k^2+\frac{1}{2}k-\frac{1}{4} = 0$$

$$= \frac{1}{4}(2k-1)(2k^2+1) = 0$$

$$= \left(k-\frac{1}{2}\right)\left(k^2+\frac{1}{2}\right)$$

$(k-j\omega_0)(k+j\omega_0)$

$= k^2+\frac{1}{2}$ should factor $\Rightarrow$

$\therefore$ Choose $k = 1/2$, then $y_R(s)$ has a pole @ $s = j\omega_0 \Rightarrow$

$$y_R = \frac{k_0 s}{s^2+\omega_0^2} + y_2(s)$$

$$\therefore\quad y(k) = y(1/2) = \frac{k^2+\frac{1}{2}k+\frac{1}{4}}{k^2+\frac{1}{2}k+1}\Bigg|_{k=1/2} = \frac{\frac{1}{4}+\frac{1}{4}+\frac{1}{4}}{\frac{1}{4}+\frac{1}{4}+1} = \frac{\frac{\cancel{3}\,1}{\cancel{4}}}{\frac{\cancel{6}\,2}{\cancel{4}}} = \frac{1}{2}$$

$$y_R(s) = \frac{1}{2}\left[\frac{k\,y(k) - s\,y(s)}{k\,y(s) - s\,y(k)}\right] = \frac{1}{2}\left[\frac{\frac{1}{2}\cdot\frac{1}{2} - s\left[\frac{s^2+\frac{1}{2}s+\frac{1}{4}}{s^2+\frac{1}{2}s+1}\right]}{\frac{1}{2}\left[\frac{s^2+\frac{1}{2}s+\frac{1}{4}}{s^2+\frac{1}{2}s+1}\right] - \frac{1}{2}s}\right]$$

$$y_R(s) = \frac{\frac{1}{2}}{\frac{1}{2}}\left[\frac{\frac{1}{4}\left(s^2+\frac{1}{2}s+1\right) - s\left(s^2+\frac{1}{2}s+\frac{1}{4}\right)}{\left(s^2+\frac{1}{2}s+\frac{1}{4}\right) - s\left(s^2+\frac{1}{2}s+1\right)}\right]$$

$$= \frac{-s^3 - s^2/4 - s/8 + 1/4}{-s^3 + s^2/2 - s/2 + 1/4}$$

$$\begin{array}{r|l} & -s^2 - \frac{3}{4}s - \frac{1}{2} \\ \hline s-\frac{1}{2} & -s^3 - s^2/4 - s/8 + 1/4 \\ & -s^3 + s^2/2 \\ & \quad -\frac{3}{4}s^2 + \frac{3}{8}s \\ & \qquad -\frac{1}{2}s + \frac{1}{4} \end{array} \qquad \begin{array}{r|l} & -s^2 - 1/2 \\ \hline s-\frac{1}{2} & -s^3 + s^2/2 - s/2 + 1/4 \\ & -s^3 + s^2/2 - s/2 + 1/4 \end{array}$$

$$y_R(s) = \frac{(s-1/2)\left(-s^2-\frac{3}{4}s-\frac{1}{2}\right)}{(s-1/2)\left(-s^2-1/2\right)} = \frac{s^2+\frac{3}{4}s+\frac{1}{2}}{s^2+\frac{1}{2}} = \frac{2k_1 s}{s^2+\frac{1}{2}} + y_2(s)$$

$$\frac{s^2+1/2}{s}\left[\frac{s^2+3/4\,s+\frac{1}{2}}{s^2+\frac{1}{2}}\right]\Bigg|_{s^2=-1/2} = 2k_1$$

$$= \frac{\left(-\cancel{\frac{1}{2}}+\frac{3}{4}s+\cancel{\frac{1}{2}}\right)}{s} = \frac{3}{4} \Rightarrow k_1 = 3/8$$

$$\therefore\ y_R(s) = \frac{\frac{3}{4}s}{s^2+\frac{1}{2}} + y_2(s) = \frac{\frac{3}{4}s}{s^2+\frac{1}{2}} + 1 \Rightarrow z(s) = \frac{1}{\frac{\frac{3}{4}}{s+\frac{1}{2s}} + 1}$$

$$s^2+\frac{3}{4}s+\frac{1}{2} = \frac{3}{4}s + \left(s^2+\frac{1}{2}\right)y_2 \Rightarrow y_2 = 1$$

$$z(s) = \frac{1}{\frac{3}{4}\left(s+\frac{1}{2s}\right)^{-1}+1}$$

$z(s) \rightarrow$