

$$1) \begin{bmatrix} C & -C & 0 \\ -C & C & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{x} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} u, \quad Y = \begin{bmatrix} 0 & 0 & 1 \\ 0 & G & -1 \end{bmatrix} x$$

a. Transform $x \Rightarrow \hat{x}$

$$\begin{aligned} sCx_1 - sCx_2 &= x_1 - x_2 \\ -sCx_1 + sCx_2 &= x_1 - x_3 - u_1 \\ 0 &= x_2 - u_2 \Rightarrow x_2 = u_2 \end{aligned}$$

$$\left. \begin{aligned} sCx_1 &= x_1 + u_2(sC-1) \\ -sCx_2 &= x_2 - x_3 - u_1 - sCu_2 \end{aligned} \right\} \Rightarrow s \begin{bmatrix} C & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & sC-1 \\ -1 & -sC \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

• Use P & Q to transform equation into desired form
 • $Q^{-1}x = \hat{x}$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} C & 0 \\ -C & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \hat{x} = I^{-1}x = x = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$$

P E Q

$$s \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix} \hat{x} = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \hat{x} + \begin{bmatrix} 0 & sC-1 \\ -1 & -1 \end{bmatrix} u \quad x_2 = u_2 = \hat{x}_2$$

$$Y = \begin{bmatrix} 0 & 1 \\ G & -1 \end{bmatrix} \hat{x}$$

b. Find $Y(s)$

$$Y(s) = C[sE - A]^{-1}B$$

$$\begin{aligned} &= \begin{bmatrix} 0 & 1 \\ G & -1 \end{bmatrix} \begin{bmatrix} sC-1 & 0 \\ -2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & sC-1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ G & -1 \end{bmatrix} \left(\frac{1}{sC-1} \right) \begin{bmatrix} 1 & 0 \\ 2 & sC-1 \end{bmatrix} \begin{bmatrix} 0 & sC-1 \\ -1 & -1 \end{bmatrix} \\ &= \frac{1}{sC-1} \begin{bmatrix} 0 & 1 \\ G & -1 \end{bmatrix} \begin{bmatrix} 0 & sC-1 \\ -sC+1 & sC-1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ G & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} -1 & 1 \\ 1 & G-1 \end{bmatrix}} \end{aligned}$$