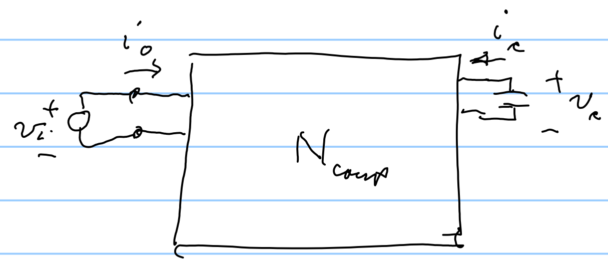
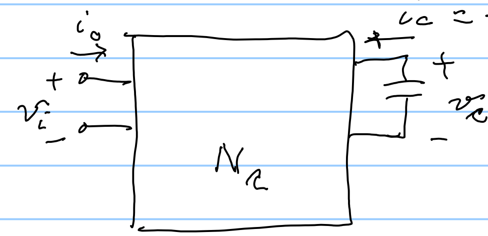
Note Title 10/31/2017

ENEE 610
10/31/17

$$\dot{x} = Ax + Bu \quad \Rightarrow \quad \dot{v}_c = A v_c + B v_i$$
$$y = Cx + Du \qquad\qquad i_o = C v_c + D v_i$$





$i_c = -C_{cap}\dot{v}_c$, choose $C_{cap} = 1$ F

$$-\dot{v}_c = \begin{bmatrix} i_c \\ i_o \end{bmatrix} = Y_{comp} \begin{bmatrix} v_c \\ v_i \end{bmatrix} = \begin{bmatrix} -A & -B \\ C & D \end{bmatrix} \begin{bmatrix} v_c \\ v_i \end{bmatrix}$$

$$Y_{comp} = \begin{bmatrix} -A & B \\ C & D \end{bmatrix}$$ a constant matrix

if A, B, C, D are constant $\Rightarrow Y_{comp} = \underbrace{\frac{1}{2}\left(Y_{comp} + Y_{comp}^T\right)}_{\text{use OTA's}} + \underbrace{\frac{1}{2}\left(Y_{comp} - Y_{comp}^T\right)}_{\text{gyrators}}$

can make with OTA's

if $x = P\hat{x}$, P^{-1} existing

$$\dot{x} = P\dot{\hat{x}} = AP\hat{x} + Bu$$
$$y = CP\hat{x} + Du$$

$$\dot{\hat{x}} = P^{-1}AP\hat{x} + P^{-1}Bu$$
$$y = CP\hat{x} + Du$$

$$\hat{Y}_{comp} = \begin{bmatrix} -P^{-1}AP & -P^{-1}B \\ CP & D \end{bmatrix} = \begin{bmatrix} P^{-1} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -A & -B \\ C & D \end{bmatrix} \begin{bmatrix} P & 0 \\ 0 & 1 \end{bmatrix}$$

there exists a P to make $\hat{Y}_{comp} = \underbrace{\frac{\hat{Y} + \hat{Y}^T}{2}}_{\text{is positive semi-definite}} + \frac{\hat{Y} - \hat{Y}^T}{2}$ positive-real matrix

by the positive-real lemma

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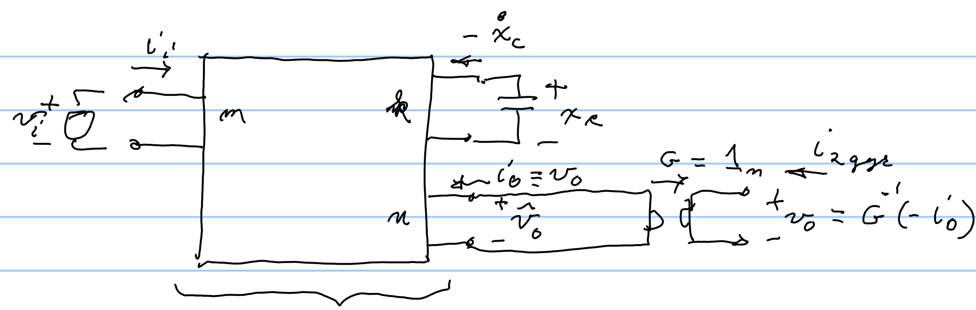
uses algebraic Riccati equations to find P

If $v_o = y$ at different ports than $u = v_i$

$$\dot{x}_o = Ax_c + Bv_i$$
$$v_o = Bx_c + Dv_i$$



B is $k \times m$

$i_i' = u$ is an m vector

x_{21} should be $m \times k$

a nice choice is B^T as

the m & k ports for B can be

made with gyrators

$$\begin{bmatrix} i_c \\ i_i' \\ i_o \end{bmatrix} = \underbrace{\begin{bmatrix} -A & -B & x_{13} \\ x_{21} & x_{22} & x_{23} \\ C & D & x_{33} \end{bmatrix}} \begin{bmatrix} v_c = x_c \\ v_i' \\ \hat{v}_o \end{bmatrix}$$

$= 0$ if no load on the gyrator ($i_{g_2} = 0$, $i'_{2gy} = -G\hat{v}_o$

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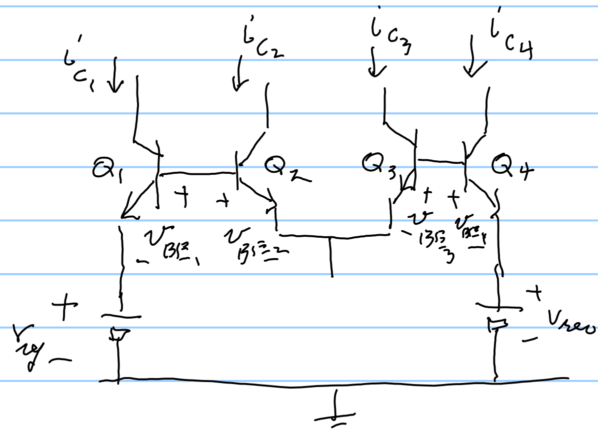
$= 0$ if $i'_{2gys} = 0$

can choose $x_{13} = -C^T$, $x_{23} = -D^T$, $x_{22} = 0_m = x_{33}$ or skew symmetric

Translinear loops



by KVL

$$0 = -\cancel{V_{ref}} - v_{BE_1} + v_{BE_2} - v_{BE_3} + v_{BE_4} + \cancel{v_{ref}}$$

$$v_{BE_1} + v_{BE_3} = v_{BE_2} + v_{BE_4}$$

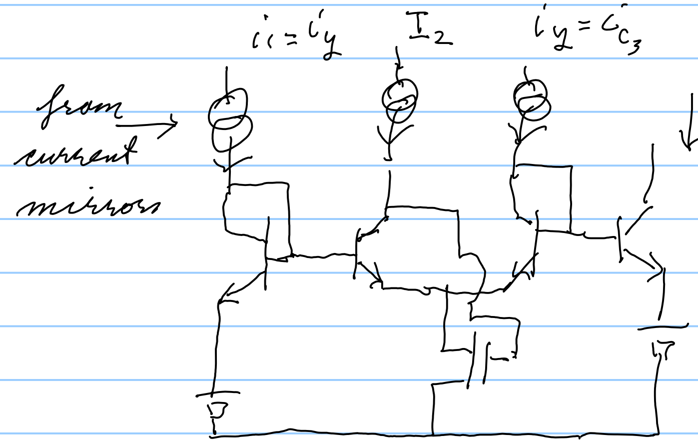
$$V_T\left(\ln\frac{i_{C_1}}{I_{O_1}} + \ln\frac{i_{C_3}}{I_{O_3}}\right) = V_T\left(\ln\frac{i_{C_2}}{I_{O_2}} + \ln\frac{i_{C_4}}{I_{O_4}}\right)$$

$$i_D = I_O\left(e^{v_D/V_T} - 1\right)$$
$$\approx I_O e^{v_D/V_T} \Rightarrow v_D = V_T \ln\left(\frac{i_D}{I_O}\right)$$

$$\ln\left(\frac{i_{C_1} \cdot i_{C_3}}{I_{O_1} I_{O_3}}\right) = \ln\left(\frac{i_{C_2} i_{C_4}}{I_{O_2} I_{O_4}}\right)$$

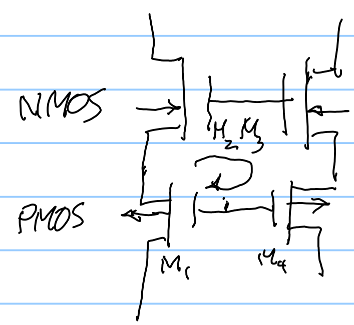
$$i_{C_1} i_{C_3} = i_{C_2} i_{C_4}\left[\frac{I_{O_1} I_{O_3}}{I_{O_2} I_{O_4}}\right]$$

$i_{c_1} = \dfrac{i_{c_2} i_{c_4}}{i_{c_3}}$, gives a means to get the product of two positive currents



$i_{out} = i_{c_4} = \dfrac{i_y^2}{I_2}$

for MOS



$0 = v_{GS_{P_1}} - v_{GS_{M_2}} + v_{GS_{M_3}} - v_{GS_{P_4}}$

gives translinear MOS loop