ENEE 610
10/26/17

Note Title 10/24/2017

$$T(s)\,T(-s) = \frac{1}{(-1)^n s^{2n} + 1} = \frac{1}{D(-s)\,D(s)}$$

desire zeroes of $s^{2n} + (-1)^n = 0 \Rightarrow s^{2n} = -(-1)^n = (-1)^{n+1}$

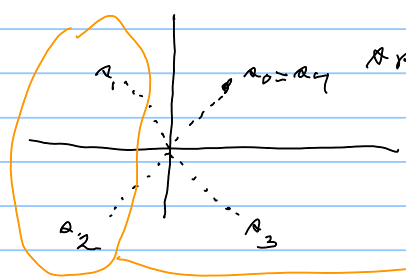
if $n =$ even, $(-1)^{n+1} = -1$, $s^{2n} = -1 = e^{j\pi} = e^{j(\pi + k \times 2\pi)}$, $k = 0, \pm 1, \pm 2, \ldots$

$$s_k = e^{j(\pi + 2k\pi)/2n} \qquad k = 0, \pm 1, \pm 2$$

$k = 0$ $\quad s_0 = e^{j\pi/2n}$

$s_1 = e^{j\frac{\pi + 2\pi}{2n}}$

if $n = 2 \Rightarrow s_0 = e^{j\pi/4}$, $s_1 = e^{j\left(\frac{\pi}{4} + \pi/2\right)}$



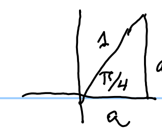
s plane $\quad e^{j\left(\frac{\pi + 8\pi}{4}\right)} = s_4$

$$\frac{1}{D(s)\,D(-s)} = \prod_{i=0}^{3} \frac{1}{(s - s_i)}$$

choose these for $T(s) = \frac{1}{D(s)} = \frac{1}{(s - s_1)(s - s_2)}$

$$T(s) = \frac{1}{\left(s - e^{j(\pi/4 + \pi/2)}\right)\left(s - e^{j(\pi/4 + 2\pi/2)}\right)}$$

$\therefore \quad T(s) = \dfrac{1}{(s + \cos\pi/4 - j\sin\pi/4)(s + \cos\pi/4 + j\sin\pi/4)}$



$a^2 + a^2 = 1$

$a = \pm 1/\sqrt{2}$

$$= \frac{1}{\left(s + \frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)\left(s + \frac{1}{\sqrt{2}} + j\,1/\sqrt{2}\right)}$$

$$= \frac{1}{s^2 + \left(\frac{1}{\sqrt{2}} - j\,1/\sqrt{2}\right)s + \left(\frac{1}{\sqrt{2}} + j\,1/\sqrt{2}\right)s + \left(\frac{1}{\sqrt{2}} - j\,1/\sqrt{2}\right)\left(1/\sqrt{2} + j\,1/\sqrt{2}\right)}$$

$$= \frac{1}{s^2 + \frac{2}{\sqrt{2}}s + \left(\frac{1}{2} + \frac{1}{2}\right)} = \frac{1}{s^2 + \sqrt{2}\,s + 1} \Longrightarrow$$

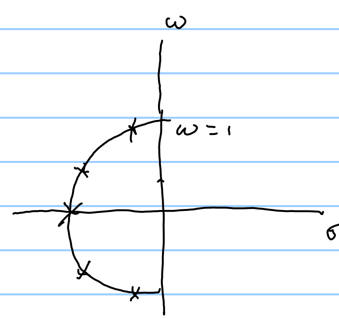
Maximally flat

degree 2 = $\delta[T(s)] = 2$

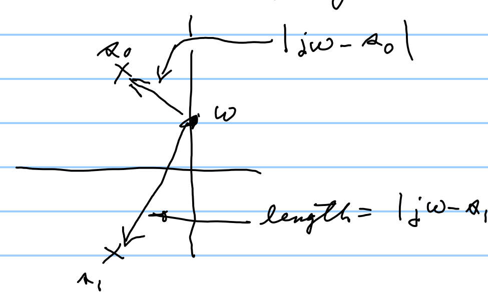
transfer function

↑

a Butterworth polynomial

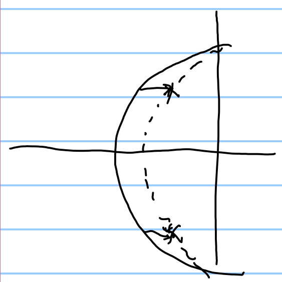


s - plane



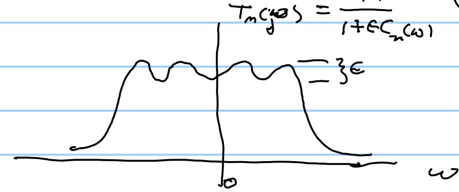
so if use an ellipse rather than a circle get equal ripple transfer functions

give Chebyshev polynomials



$$C_n(x) = \cos(n \arccos x) \quad \text{if } |x|<1$$
$$\cosh(n \operatorname{arccosh} x) \quad \text{if } |x|>1$$

$$T_n(j\omega) = \frac{1}{1+\epsilon C_n(\omega)}$$



given $T(s)$ find a circuit $\Rightarrow$ $T(s) = \frac{N(s)}{D(s)}$, $\delta[N] \leq \delta[D]$

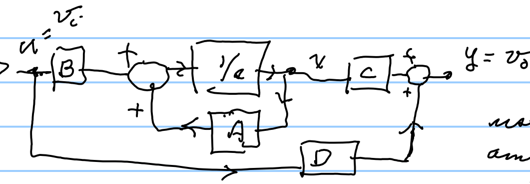
then we can factor the polynomials N & D (with real coefficients) into degree one or degree two factors

$$T(s) = \frac{ds^2 + es + f}{s^2 + a_1 s + a_0} \approx d + \frac{gs + h}{s^2 + a_1 s + a_0}$$

If we have $\dot{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^{\cdot} = Ax + Bu$, $y = Cx + Du$, $u = v_i$, $y = v_o$

$\Rightarrow sx = Ax + Bu$

$\Rightarrow x = \frac{1}{s}\left[Ax + Bu\right] \Rightarrow$



use RC & op-amp can do with the TI UAF
"
universal active filter

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u \Leftarrow sx = Ax + Bu$$

$$y = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + du$$

$$sI_2 x - Ax = Bu$$
$$x = (sI_2 - A)^{-1} B u$$

$$(sI_2 - A)^{-1} = \begin{bmatrix} s-0 & -1 \\ a_0 & s+a_1 \end{bmatrix}^{-1} = \frac{1}{s(s+a_1)+a_0} \begin{bmatrix} s+a_1 & 1 \\ -a_0 & s \end{bmatrix}$$

$$\frac{y}{u} = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \frac{1}{s^2 + a_1 s + a_0} \begin{bmatrix} s+a_1 & 1 \\ -a_0 & s \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + d$$

$$= \frac{\begin{bmatrix} c_1 & c_2 \end{bmatrix}}{s^2 + a_1 s + a_0} \begin{bmatrix} b_1 s + a_1 b_1 + b_2 \\ -a_0 b_1 + s b_2 \end{bmatrix} + d = \frac{\left[c_1 b_1 s + c_1(a_1 b_1 + b_2) + c_2(-a_0 b_1 + s b_2)\right]}{s^2 + a_1 s + a_0} + d$$

$$= \frac{\left[(c_1 b_1 + c_2 b_2)s + (c_1 a_1 b_1 + c_1 b_2 - c_2 a_0 b_1)\right]}{s^2 + a_1 s + a_0} + d$$

$$= \frac{gs + h}{s^2 + a_1 s + a_0} + d \qquad \begin{aligned} g &= c_1 b_1 + c_2 b_2 \\ h &= c_1 a_1 b_1 + c_1 b_2 - c_2 a_0 b_1 \end{aligned}$$

try $b_2 = 0 \Rightarrow \quad c_1 b_1 = g, \quad c_1 a_1 b_1 - c_2 a_0 b_1 = h \leftarrow$ choose $c_2 = \dfrac{h - a_1 c_1 b_1}{-a_0 b_1}$

allows active filter design, LP can use Butterworth or Chebyshev polynomials