Note Title 10/24/2017

ENEE 610
10/24/17

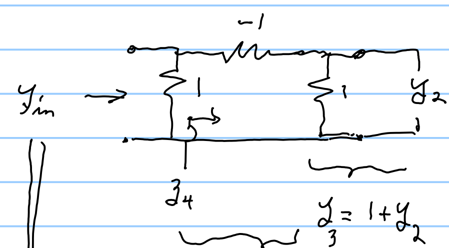
Wohlers' $S(s)$ syntheses, S 1×1 & rational with real coefficients

$$S(s) = n(s)/d(s) = \frac{n(s)/d_1(s)}{d(s)/d_1(s)} = \frac{S_1(s)}{S_2(s)}$$, S_1 & S_2 bounded real, BR

choose $d_1(s)$ Hurwitz (no zeroes in $\sigma \geq 0$), $\delta[d_1] \geq$ max of $\delta[n]$ or $\delta[d]$

Then to synthesize $S(s)$, synthesize S_1 & S_2

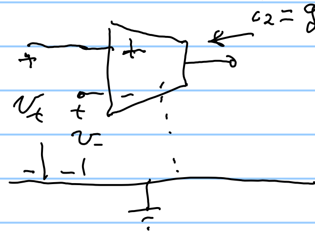
to create S_2:



$$\mathcal{Z}_4 = -1 + \frac{1}{1+y_2} = \frac{-1-y_2+1}{1+y_2} = \frac{-y_2}{1+y_2}$$

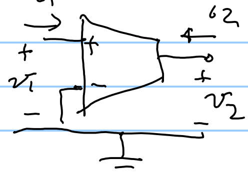
$$= 1 + \frac{1}{\mathcal{Z}_4} = 1 + \frac{1}{-y_2/(1+y_2)} = 1 - \frac{(1+y_2)}{y_2} = \frac{y_2 - 1 - y_2}{y_2} = \frac{-1}{y_2}$$

$S = (1+y)^{-1}(1-y)$; $S_{in} = \left(1+\left(-\frac{1}{y_2}\right)\right)^{-1}\left(1-\left(-\frac{1}{y_2}\right)\right) = \frac{(y_2+1)}{(y_2-1)} = -\left(\frac{1+y_2}{1-y_2}\right) = -\frac{1}{S_2}$



$i_2 = g_m(v_+ - v_-)$

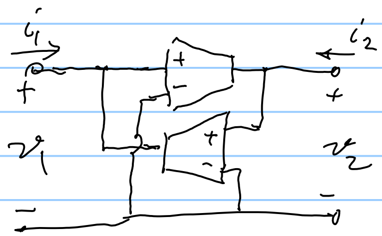
$=$



$i_1 = 0$

$Y_{OTA} = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix}$

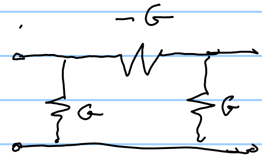
OTA = operational transconductance amplifier



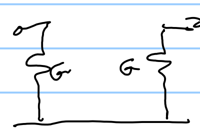
identical OTA's

$\Rightarrow Y = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix} + \begin{bmatrix} 0 & g_m \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & g_m \\ g_m & 0 \end{bmatrix}$

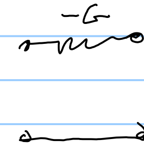
active
$P_{in} < 0$
for $\neq 0$ input



$\Rightarrow Y_1 \Rightarrow$

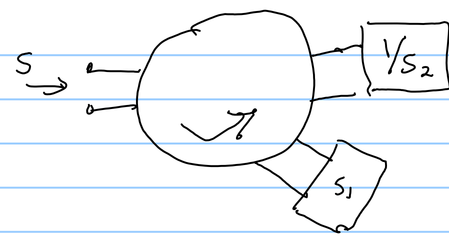


$+ \; Y_2 \Rightarrow$



$\Rightarrow Y = \begin{bmatrix} 0 & +G \\ +G & 0 \end{bmatrix}$

$Y_1 = \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix}$ $\quad Y_2 = \begin{bmatrix} -G & -(-G) \\ -(-G) & -G \end{bmatrix}$



$S = \frac{1}{S_2} \cdot S_1$ gives a not BR S by two BR S's

$\Rightarrow$ go to y or z for S_1, S_2 & use Bott-Duffin synthesis

Example

$S = s$ is not BR $\quad 1-(j\omega)^*(j\omega) = 1-\omega^2 \Rightarrow |1-\omega^2| \to \infty$ as $\omega \to \infty$

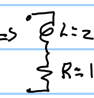
$= s/1$

$= \frac{s/d_1}{1/d_1}$ $\quad d_1 =$ Hurwitz, $\delta[n] = \delta[s] = 1$, $\delta[d] = \delta[1] = 0$

choose $d_1 = s+1$

$= \frac{s/(s+1)}{1/(s+1)}$ $\quad \Rightarrow S_1 = \frac{(1-y_1)}{(1+y_1)} \Rightarrow y_1 = \frac{(1-S_1)}{(1+S_1)} = \frac{1-\frac{s}{s+1}}{1+s/s+1} = \frac{s+1-s}{s+1+s}$

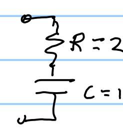
$= \frac{1}{2s+1} \Rightarrow z_1 = 2s+1 \Rightarrow$

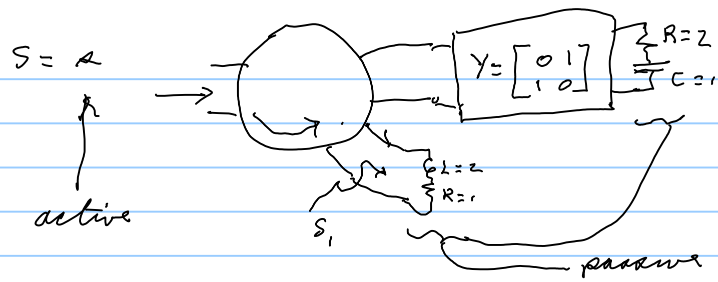


$z_2 S_2 + S_2 = z_2 - 1$ $\qquad S_2 = \frac{z_2 - 1}{z_2 + 1}$

$z_2(S_2 - 1) = -S_2 - 1$

$z_2 = \frac{S_2+1}{1-S_2} = \frac{(1/s+1)+1}{1-(\frac{1}{s+1})} = \frac{1+s+1}{s+1-1} = \frac{2s+1}{s} = 2+\frac{1}{s}$



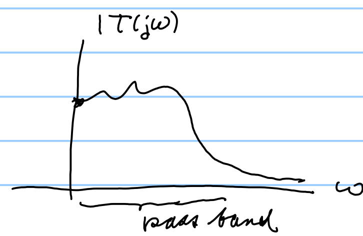


For matrix $S = \begin{bmatrix} m_{11}/d & m_{12}/d \\ m_{21}/d & m_{22}/d \end{bmatrix} = \underbrace{\begin{bmatrix} d/d_1 & 0 \\ 0 & d/d_1 \end{bmatrix}^{-1}}_{\substack{\text{uses 2 negative} \\ \text{resistors or 4 OTA's} \\ \text{to get } S_2^{-1} \text{ given } S_2}} \begin{bmatrix} m_{11}/d_1 & m_{12}/d_1 \\ m_{21}/d_1 & d_{22}/d_1 \end{bmatrix} \Rightarrow$ product of S_2^{-1} & S_1

both S_1 & S_2 are BR

∴ by Wohlers' scheme can synthesize "almost" any active circuit
M. Ronald

approximation: low pass



Maximally flat low pass $T(s) = \dfrac{1}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$ can assume $a_0 = 1$ also

$$\left.\frac{d^m |T(j\omega)|}{d\omega^m}\right|_{\omega=0} = 0 \quad \text{for as many integers } m > 0 \text{ as possible}$$

$$\left. T(-s)\,T(s) \right|_{s=j\omega} = |T(j\omega)|^2 \; ; \quad \left.\frac{d|T(j\omega)|^2}{d\omega} = 2\underbrace{|T(j\omega)|}_{\neq 0 @ \omega=0} \frac{d|T(j\omega)|}{d\omega}\right|_{\omega=0}$$

$\therefore$ to force $\dfrac{d|T(j\omega)|}{d\omega} = 0$ can force $\dfrac{d}{d\omega}|T(j\omega)|^2 \Rightarrow 0$

$$\frac{d \frac{1}{|T(j\omega)|}}{d\omega} = -\frac{d|T(j\omega)|}{d\omega} \times \frac{1}{\underbrace{|T(j\omega)|^2}_{\neq 0 @ \omega=0}} \Rightarrow 0 \text{ implies } \frac{d|T(j\omega)|}{d\omega} \Rightarrow 0$$

$\therefore$ can force $\dfrac{d^m \frac{1}{|T(j\omega)|^2}}{d\omega^m} = 0 @ \omega = 0$

But $\dfrac{1}{T(s)} = s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + 1 \Rightarrow \dfrac{1}{T^*(j\omega)\,T(j\omega)} = \dfrac{1}{T(-j\omega)\,T(j\omega)}$

$$\frac{1}{T(-j\omega)T(j\omega)} = \left((j\omega)^n + \cdots + 1\right)\left((-j\omega)^n + \cdots + 1\right) = \left((j)^n(-j)^n(\omega^{2n}) + \cdots\right.$$

$$= \omega^{2n} + b_1\omega^{2n-2} + \cdots + 1$$

$$= f(\omega^2)$$

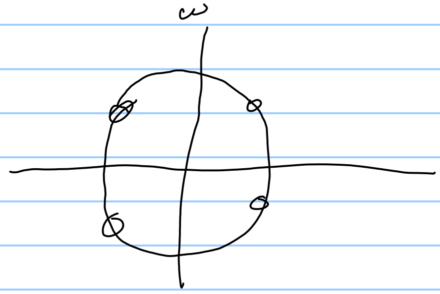
$$\frac{d\,\frac{1}{T(-j\omega)T(j\omega)}}{d\omega} = 2n(\omega^{2n-1}) + b_1(\omega^{2n-3})(2n-2) + \cdots \qquad 2b_0\omega^{2-1}$$

keep differentiating & set to 0 @ $\omega = 0 \Rightarrow$ all these coefficients $b_0, \ldots = 0$

$$\therefore \frac{1}{T(-j\omega)T(j\omega)} = \omega^{2n} + 1 \Rightarrow \frac{1}{T(-s)T(s)} \qquad \text{set } s = \omega/j \Rightarrow \text{set } \omega = js$$

$$\frac{1}{T(-s)T(s)} = (js)^{2n} + 1 = (-1)^n \cdot s^{2n} + 1$$

ω



σ

$s = \sigma + j\omega$

desire to factor $(-1)^n s^{2n} + 1$ into a Hurwitz polynomial $\times$ Hurwitz conjugate polynomial

$(-1)^n s^{2n} = -1$ gives the zeros of $\frac{1}{T(-s)T(s)}$

$$= \frac{1}{T_* T}$$