ENEE 610
10/19/17

Note Title 10/16/2017

Sensitivity $S_x^{T(s,x)} = \frac{x}{T}\frac{\partial T}{\partial x}$, if $T(j\omega) = |T(j\omega)| e^{j\angle T}$

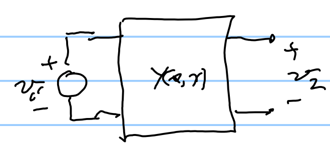
then $S_x^{T(j\omega)} = \frac{x}{|T(j\omega)| e^{j\angle T}} \cdot \frac{d\, |T(j\omega)| e^{j\angle T}}{dx}$

$= \frac{x}{|T| e^{j\angle T}} \cdot \left[\frac{d|T(j\omega)|}{dx} e^{j\angle T} + |T(j\omega)|\, d\, e^{j\angle T} \right]$

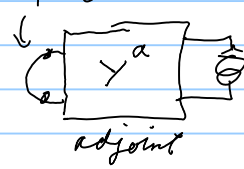
$= \frac{x}{|T(j\omega)|} \cdot \frac{d\,|T(j\omega)|}{dx} + \frac{x}{e^{j\angle T}} \cdot e^{j\angle T} \cdot \left(j \frac{d\angle T}{dx}\right)$

$= S_x^{|T(j\omega)|} + j\, x \frac{d\angle T}{dx}$

assume



$\Longrightarrow$



desire $S_x^{v_2/v_i}$

assume both have the same graph $v^T i = 0 \Rightarrow v_b^T \cdot i_b^a - v_b^{aT} i_b$

adjoint

original

$$v_b = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_{b-2} \end{bmatrix} \qquad i = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_{b-2} \end{bmatrix}$$

, also for adjoints (put a super a on these 2s)

inside the 2-port

$$v_b^T i_b^a - v_b^{aT} i_b = \begin{bmatrix} v_1 & v_2 & v_{b-2}^T \end{bmatrix} \begin{bmatrix} i_1^a \\ i_2^a \\ i_{b-2}^a \end{bmatrix} - \begin{bmatrix} v_1^a & v_2^a & v_{b-2}^{aT} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_{b-2} \end{bmatrix} = v_1 i_1^a - v_1^a i_1 + v_2 i_2^a - v_2^a i_2 + v_{b-2}^T \left[Y^a v_{b-2}^a \right] - \underbrace{v_{b-2}^{aT} \left[Y(a,x)\, v_{b-2} \right]}_{\text{transpose}}$$

$$v_1 i_1^a - v_1^a i_1 + v_2 i_2^a - v_2^a i_2 + v_{b-2}^T \left[Y^a - Y^T \right] v_{b-2}^a = 0$$

fix x in adjoint circuit so the adjoint circuit has no changes when x changes

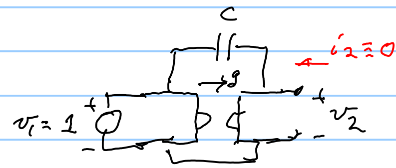
fix $v_1(a)$ ⇒ now take derivatives of all these terms

$$\underset{0}{\underset{\parallel}{\frac{d(v_1 i_1^a)}{dx}}} - \underset{0}{\underset{\parallel}{\frac{dv_1^a i_1}{dx}}} + \underset{\frac{dv_2}{dx} \cdot i_2^a}{\underset{\parallel}{\frac{d(v_2 i_2^a)}{dx}}} + \frac{d v_{b-2}^T}{dx} \underbrace{\left[Y_{b-2}^a - Y_{b-2}^T \right]}_{\substack{\text{force} = 0 \\ \text{by } Y_{b-2}^a = Y_{b-2}^T}} v_{b-2}^a + \underset{0}{\underset{\parallel}{v_{b-2}^T \left[\frac{dY_{b-2}^a}{dx} - \frac{dY_{b-2}^T}{dx} \right] v_{b-2}^a}} + \underset{0}{\underset{\parallel}{v_{b-2}^T \left[Y_{b-2}^a - Y_{b-2}^T \right] \frac{dv_{b-2}^a}{dx}}} = 0$$

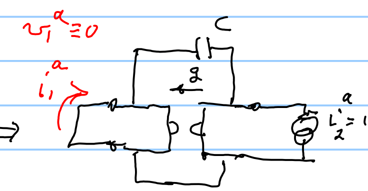
$$= v_2$$

choose $v_1 = 1$ then $v_2 = T(a,x)$ ⇒ $i_2^a \cdot \frac{dT(a,x)}{dx} = -v_{b-2}^T \left(-\frac{dY_{b-2}^T}{dx} \right) \cdot v_{b-2}^a$

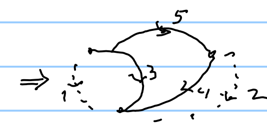
choose $i_2^{a} = 1 \Rightarrow \frac{dT(a,x)}{dx} = \underbrace{\frac{dv_2}{dx} = + v_{b-2}^{T} \left(\frac{dY_{b-2}}{dx}\right)^{T} \cdot v_{b-2}^{a}}$



$\Rightarrow$



$\Rightarrow$



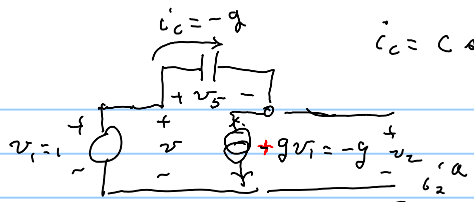
derive $S_c^{v_2/v_1} \Rightarrow \underset{\text{2-port}}{Y} = \begin{bmatrix} ac & -ac+g \\ -ac-g & ac \end{bmatrix} \Rightarrow \underset{\text{2-port}}{Y^{T}} = \begin{bmatrix} ac & -ac-g \\ -ac+g & ac \end{bmatrix}$

$\frac{dY^{T}_{\text{2-port}}}{dc} = \begin{bmatrix} a & -a \\ -a & a \end{bmatrix}$ but need the branch by branch Y not the 2-port Y

$\begin{bmatrix} i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0 & g & 0 \\ -g & 0 & 0 \\ 0 & 0 & ac \end{bmatrix} \begin{bmatrix} v_3 \\ v_4 \\ v_5 \end{bmatrix}$, $Y_{b-2} = \begin{bmatrix} 0 & g & 0 \\ -g & 0 & 0 \\ 0 & 0 & ac \end{bmatrix} \Rightarrow Y_{b-2}^{T} = \begin{bmatrix} 0 & -g & 0 \\ g & 0 & 0 \\ 0 & 0 & ac \end{bmatrix} \Rightarrow \frac{dY_{b-2}^{T}}{dc} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a \end{bmatrix}$

$\frac{dv_2}{dc} = \begin{bmatrix} v_3 & v_4 & v_5 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a \end{bmatrix} \begin{bmatrix} v_3^{a} \\ v_4^{a} \\ v_5^{a} \end{bmatrix} = \begin{bmatrix} 0 & 0 & a v_5 \end{bmatrix} \begin{bmatrix} v_3^{a} \\ v_4^{a} \\ v_5^{a} \end{bmatrix} = a\, v_5 \cdot v_5^{a}$

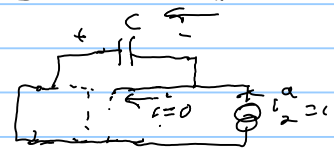
$=$



$i_c = C\alpha \cdot v_5 \Rightarrow v_5 = \frac{1}{C\alpha} \cdot (+g)$

$v_2 = 1 - v_5 = 1 + \frac{g}{C\alpha} = \frac{C\alpha + g}{C\alpha}$

for adjoint



$v_5^a = -\frac{1}{C\alpha} \cdot 1$

$\Rightarrow \frac{dv_2}{dC} = \frac{d\overset{v_2}{T}}{dC} = \frac{d\,T(\alpha, c)}{dC} = \alpha \cdot \frac{1}{C\alpha}(+g)\left(\frac{-1}{C\alpha}\right)$

$= -g / C^2\alpha$

should agree

$S_\alpha^{T(\alpha)} = \frac{C\, dT/dC}{T(\alpha)} = \frac{C}{\underset{\alpha C}{C\alpha + g}} \cdot \frac{g}{C^2\alpha} = \frac{g}{(C\alpha + g)}$

as a check: $\frac{dv_2}{dC} = \frac{d}{dC}\left(1 + \frac{g}{C\alpha}\right) = \frac{-g}{(\alpha C)^2} \cdot \alpha = S_x^T$