Note Title 10/12/2017

Example of Bott-Duffin, given $y(s)$, PR

$$y(s) = \frac{4s^2 + s + 13/16}{s^2 + \frac{1}{4}s + 1/4}$$

desire $\min_{\omega \geq 0} \operatorname{Re} y(j\omega) = G$, $y(s) - G$ still PR

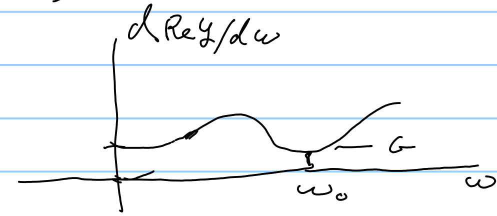
$\operatorname{Re}\left(y(s) - G\right)\Big|_{s = j\omega_0} = 0$; $\operatorname{Im}\left(y(j\omega_0) - G\right) \neq 0$

$$\operatorname{Re} y(j\omega) = \left[y(j\omega) + y(-j\omega)\right]/2$$

$$y(j\omega) = \frac{(-4\omega^2 + 13/16) + j\omega}{(-\omega^2 + 1/4) + j\omega/4} \times \frac{(-\omega^2 + 1/4) - j\omega/4}{(-\omega^2 + 1/4) - j\omega/4} =$$

$$\operatorname{Re} y(j\omega) = \frac{(-4\omega^2 + 13/16)(-\omega^2 + 1/4) + \omega^2/4}{(-\omega^2 + 1/4)^2 + (\omega/4)^2 = D(\omega)} = \frac{\left[(4\omega^4 - (13/16 + 1)\omega^2 + 13/64 + \omega^2/4\right]}{D(\omega)}$$

$$\left.\frac{d \operatorname{Re} y(j\omega)}{d\omega}\right|_{\omega_0} = 0 \quad ; \quad \operatorname{Re} y(j\omega_0) = G$$



here $G = 3$

here $y(\lambda) - 3 = \dfrac{4\lambda^2 + \lambda + 13/16}{\lambda^2 + \frac{1}{4}\lambda + 1/4} - 3 = \dfrac{\lambda^2 + \frac{1}{4}\lambda + 1/16}{\lambda^2 + \frac{1}{4}\lambda + 1/4} = y_1$

$$y_1(j\omega) = \frac{(-\omega^2 + 1/16) + \frac{1}{4} j\omega}{(-\omega^2 + 1/4) + 1/4\, j\omega} \times \frac{(-\omega^2 + 1/4) - j\omega/4}{(-\omega^2 + 1/4) - j\omega/4}$$

Re $y_1(j\omega) = \dfrac{(-\omega^2 + 1/16)(-\omega^2 + 1/4) + \omega^2/16}{(-\omega^2 + 1/4)^2 + (\omega/4)^2} = \dfrac{\omega^4 - (\frac{1}{16} + 1/4 - 1/16)\omega^2 + \frac{1}{4\times16}}{D(\omega)}$

$= \dfrac{\omega^4 - \frac{1}{4}\omega^2 + \frac{1}{4\times16}}{D(\omega)}$, $\Rightarrow 0$ @ $\omega_0^2 = +1/8 \pm \frac{1}{2}\sqrt{(1/4)^2 - 4 \times \frac{1}{4\times16}}$

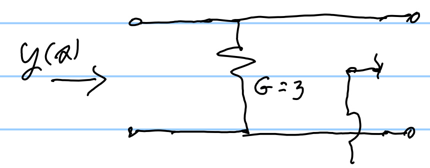
$\omega_0^2 = 1/8$, $\omega_0 = \pm \frac{1}{2\sqrt{2}}$

$$y_1(j\omega_0) = \frac{-\frac{1}{8} + \frac{1}{16} + j\frac{1}{8\sqrt{2}}}{-\frac{1}{8} + 1/4 + j\, 1/8\sqrt{2}} = \frac{\frac{1}{8}\left(-\frac{1}{2} + j\frac{1}{\sqrt{2}}\right)}{1/8\,(1 + j\, 1/\sqrt{2})} \qquad \lambda_0 = j\omega_0$$

$$= \frac{(-1/2 + j\, 1/\sqrt{2})}{(1 + j\, 1/\sqrt{2})} \times \frac{(1 - j\, 1/\sqrt{2})}{(1 - j\, 1/\sqrt{2})} = \frac{(-1/2 + 1/2) + j\left[1/\sqrt{2} + \frac{1}{2\sqrt{2}}\right]}{1^2 + (1/\sqrt{2})^2} = \frac{2}{3} \times \frac{1}{2\sqrt{2}}\left[2 + 1\right] j$$

$= (1/\sqrt{2})\, j$

$\therefore$ desire to cancel this term with the Richards functio



$y_1(s) = \text{minimum}$

@ $\omega_0 = 1/2\sqrt{2}$ $\quad \operatorname{Re} y_1(s)\Big|_{s=j\omega_0} = 0$, $\operatorname{Im} y_1(s)\Big|_{s=j\omega_0} = 1/\sqrt{2}$

$y_R(s) = \cdot$ $\qquad y_i = y_1(s) = \dfrac{s^2 + \frac{1}{4}s + \frac{1}{16}}{s^2 + \frac{1}{4}s + \frac{1}{4}}$

$$y_L(s) = y_i(s)\left[\frac{k\,y_i(k) - s\,y_i(s)}{k\,y_i(s) - s\,y_i(k)}\right]$$

$$= y_1(k)\left[\frac{k\,\dfrac{k^2+\frac{1}{4}k+\frac{1}{16}}{k^2+\frac{1}{4}k+\frac{1}{4}} - s\left(\dfrac{s^2+\frac{1}{4}s+\frac{1}{16}}{s^2+\frac{1}{4}s+\frac{1}{4}}\right)}{k\left(\dfrac{s^2+\frac{1}{4}s+\frac{1}{16}}{s^2+\frac{1}{4}s+\frac{1}{4}}\right) - s\left(\dfrac{k^2+\frac{1}{4}k+\frac{1}{16}}{k^2+\frac{1}{4}k+\frac{1}{4}}\right)}\right]\Bigg|_{s=j\omega_0}$$

& solve for k

$y_R = 0$ or $1/y_{12} = 0$

$$Y_L(j\omega_0) = Y_1(k)\left[\frac{k\left(k^2+\frac{1}{4}k+\frac{1}{16}\right) - j\frac{1}{2\sqrt{2}}\cdot j\frac{1}{\sqrt{2}}\left(k^2+\frac{1}{4}k+\frac{1}{4}\right)}{k\,j\frac{1}{\sqrt{2}}\left(k^2+\frac{1}{4}k+\frac{1}{4}\right) - j\frac{1}{2\sqrt{2}}\left(k^2+\frac{1}{4}k+\frac{1}{16}\right)}\right] = \frac{\quad}{\frac{j}{\sqrt{2}}\left\{k^3+\frac{1}{4}k^2+\frac{k}{4}-\frac{k^2}{2}-\frac{k}{8}-\frac{1}{32}\right\}}$$

$\Rightarrow$ desire the denominator of this $= 0 \Rightarrow k^3 - \frac{1}{4}k^2 + \frac{1}{8}k - \frac{1}{32} = 0$

$$= (k^2+1/8)(k-1/4)$$

$(k-j\omega_0)(k+j\omega_0) = k^2+\omega_0^2 = k^2+1/8$ should factor

$$\begin{array}{r|l} & k - 1/4 \\ \hline k^2+1/8 & k^3 - \frac{1}{4}k^2 + \frac{1}{8}k - 1/32 \\ & k^3 \qquad\qquad + 1/8\,k \\ \hline & \quad -1/4\,k^2 \qquad -1/32 \\ & \quad -1/4\,k^2 \qquad -1/32 \end{array}$$

$\therefore$ choose $k = 1/4$ then $Y_L(s) = Y_R(s)$ has a pole @ $s = j\omega_0 \Rightarrow Y_R = \frac{k_0 s}{s^2+\omega_0^2} + Y_2(s)$

$\delta[Y_2] = \delta[Y_L] + 2$

$\therefore\ Y_1(k) = Y_1(1/4) = \left.\frac{k^2+\frac{1}{4}k+1/16}{k^2+1/4\,k+1/4}\right|_{k=1/4} = \frac{\frac{1}{16}+\frac{1}{16}+\frac{1}{16}}{\frac{1}{16}+1/16+1/4} = \frac{3/16}{6/16} = 1/2$

$$Y_L(s) = \frac{1}{2}\left[\frac{k\,Y_1(k) - s\,Y(s)}{k\,Y(s) - s\,Y(k)}\right] = \frac{1}{2}\left[\frac{\frac{1}{4}\cdot\frac{1}{2} - s\,Y(s)}{\frac{1}{4}Y(s) - \frac{1}{2}s}\right] = \frac{1}{2}\left[\frac{\frac{1}{8} - s\left[\frac{s^2 + \frac{s}{4} + 1/16}{s^2 + s/4 + 1/4}\right]}{\frac{1}{4}\left[\frac{s^2 + s/4 + 1/16}{s^2 + s/4 + 1/4}\right] - \frac{1}{2}s}\right]$$ is PR with a pole $s^2 + \omega_0^2 = 0$

next is rationalize cancel, pull out pole & use the Bott-Duffin bridge