ENEE 610
10/10/17

Note Title 10/10/2017

$$y_L(\alpha) = y_i(\kappa)\left[\frac{\kappa y_i(\kappa) - \alpha y_i(\alpha)}{\kappa y_i(\alpha) - \alpha y_i(\kappa)}\right]$$

$$y(\alpha) = \frac{2\alpha(\alpha^2+2)}{(\alpha^2+1)}$$

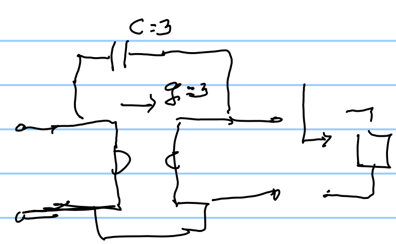
let $\kappa = 1$

$$y(\kappa) = y(1) = \frac{2(3)}{2} = 3 \qquad g = y(\kappa) = 3, \quad c = \frac{y(\kappa)}{\kappa} = \frac{3}{1}$$

$$y_L(\alpha) = 3\left[\frac{1\cdot 3 - \alpha\left(\frac{2\alpha(\alpha^2+2)}{\alpha^2+1}\right)}{1\cdot \cancel{\alpha}\left(\frac{2\alpha(\alpha^2+2)}{\alpha^2+1}\right) - \alpha\cdot 3}\right] = 3\left[\frac{3(\alpha^2+1) - 2\alpha^2(\alpha^2+2)}{2\alpha^{\cancel{2}}(\alpha^2+2) - 3\alpha(\alpha^2+1)}\right] = \frac{(\alpha+1)(\alpha-1)}{(\alpha+1)(\alpha-1)}, \ldots$$

$$= 3\left[\frac{-2\alpha^4 - 4\alpha^2 + 3\alpha^2 + 3}{2\alpha^3 + 4\alpha^{\cancel{2}} - 3\alpha^3 - 3\alpha}\right] = \frac{3\left[(\alpha^2-1)(-2\alpha^2-3)\right]}{\left[(\alpha^2-1)(-\alpha)\right]} = 3\left(\frac{2\alpha^2+3}{\alpha}\right) \qquad \text{LPR}$$

$$\begin{array}{r|l} & -2\alpha^2 - 3 \\ \hline \alpha^2 - 1 & -2\alpha^4 - \alpha^2 + 3 \\ & +2\alpha^2 \\ \hline & -3\alpha^2 \\ & -3\alpha^2 + 3 \\ \hline & 0 \end{array} \qquad \begin{array}{r|l} & -\alpha \\ \hline \alpha^2 - 1 & -\alpha^3 + \alpha \\ & -\alpha + \alpha \end{array}$$



$\delta[y] = 3 \qquad y = \frac{2\alpha(\alpha^2+2)}{\alpha^2+1}$

$$y_L = 3\left(\frac{2\alpha^2+3}{\alpha}\right); \quad \delta[y_L] = 2$$

Lossy synthesis of PR function $Y(s) \Rightarrow$ Bott-Duffin

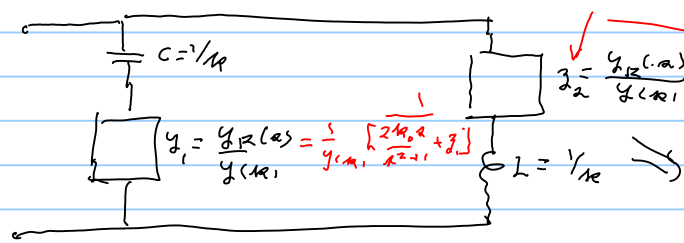
$$Y_R(s) = Y_i(s)\left[\frac{kY_i(k) - sY_i(s)}{kY_i(s) - sY_i(k)}\right] \Rightarrow Y_R\left[kY(s) - sY(k)\right] = Y(k)^2 k - sY(k)\,Y(s)$$

$$Y(s)\left[kY_R + sY(k)\right] = sY(k)Y_R + kY^2(k)$$

$$Y(s) = Y(k)\left[\frac{sY_R(s) + kY(k)}{kY_R(s) + sY(k)}\right]$$

$$\Rightarrow \frac{Y(s)}{Y(k)} = \frac{sY_R(s)}{kY_R(s) + sY(k)} + \frac{kY(k)}{kY_R(s) + sY(k)} = \frac{1}{\frac{k}{s} + \frac{Y(k)}{Y_R(s)}} + \frac{1}{\frac{Y_R(s)}{Y(k)} + s/k}$$

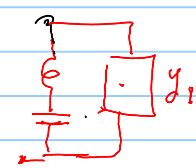
$\frac{Y(s)}{Y(k)} \rightarrow$

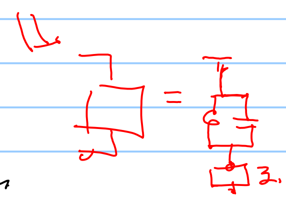


$$y_1 = \frac{Y_R(s)}{Y(k)} = \frac{1}{Y(k)}\left[\frac{2k_0 s}{s^2+1} + z_1\right]$$

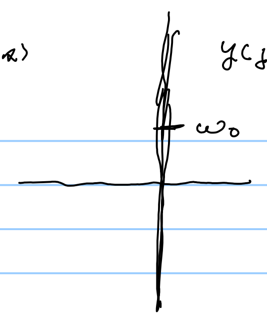
an impedance

$$z_2 = \frac{Y_R(s)}{Y(k)} = \frac{1}{Y(k)}\left[\frac{2k_0 s}{s^2+\omega_0^2} + y_1(s)\right] \leftarrow \text{an admittance}$$





P.361
Bott-Duffin
uses no transformers

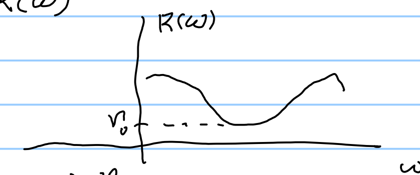


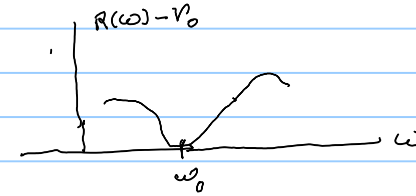
$y(j\omega) = R(\omega) + j\,I(\omega)$, $R(\omega) \geq 0 \cdot \quad -\infty < \omega < \infty$

as $y(s)$ is PR

(by max modulus theorem)

find $\min\limits_{\omega} R(\omega)$





$y(s) - r_0 =$ is still PR & $y(j\omega_0) = j\,I(\omega_0)$

$y_0(s) = y(s) - r_0$ is PR with $y_0(j\omega_0) = j\,I(\omega_0)$

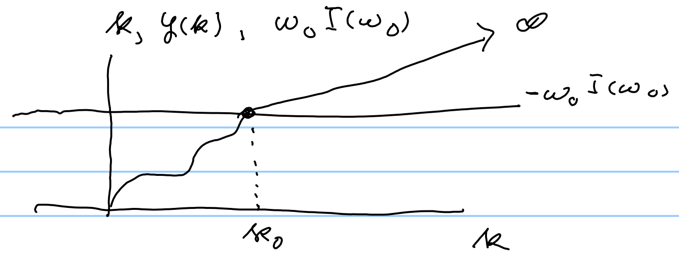
$= \text{minimum}$

∴ form y_R for a minimum function $y_0(s)$

$$y_R(s) = y_i(k)\left[\frac{k\,y_i(k) - s\,y_i(s)}{k\,y_i(s) - s\,y_i(k)}\right]\Bigg|_{s=j\omega_0} \Rightarrow y_R(j\omega_0) = y_0(k)\left[\frac{k\,y_0(k) - j\omega_0 \cdot j\,I(\omega_0)}{k\,j\,I(\omega_0) - j\,\omega_0\,y_0(k)}\right] = \frac{y_0(k)}{j}\left[\frac{k\,y_0(k) + \omega_0\,I(\omega_0)}{k\,I(\omega_0) - \omega_0\,y_0(k)}\right]$$

If $I(\omega_0) < 0$ then force numerator to be zero by a real positive k

such exists as $y_0(k)$ has no poles in k & is ∴ bounded while $0 < k < \infty$



Then @ $s_0 = j\omega_0$, $Y_R(s_0) = 0 \Rightarrow (s - j\omega_0)$ is a factor as Y_R is polynomial over polynomial

Then $\frac{1}{Y_R(s)}$ has a pole at $s = j\omega_0$, & @ $s = -j\omega_0$

$$\frac{1}{Y_R(s)} = \frac{2k_0 s}{s^2 + \omega_0^2} + Z_1(s), \quad Z_1(s) \text{ is PR}$$