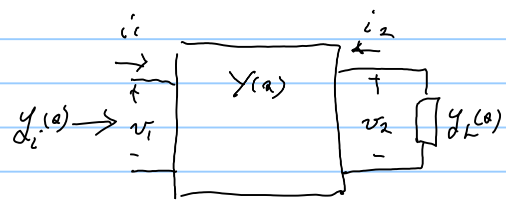
Note Title 10/4/2017

ENEE 610
10/05/17

Richards' function: p.361  $R(s) = \dfrac{k\,Z(s) - s\,Z(k)}{k\,Z(k) - s\,Z(s)}$




$i_1 = y_{11} v_1 + y_{12} v_2$

$i_2 = y_{21} v_1 + y_{22} v_2 = -y_L v_2 \Rightarrow -(y_{22} + y_L) v_2 = y_{21} v_1$

$v_2 = \dfrac{-1}{y_{22} + y_L} \cdot y_{21} v_1$

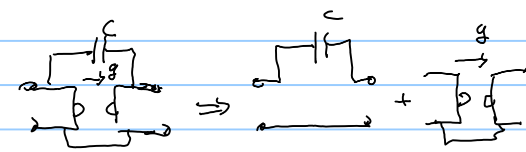
$\Rightarrow i_1 = \left[ y_{11} + y_{12} \cdot \dfrac{-1}{y_{22}+y_L} y_{21} \right] v_i \Rightarrow y_i = \dfrac{y_{11} y_{22} + y_{11} y_L - y_{12} y_{21}}{y_{22} + y_L} = \dfrac{\Delta_y + y_{11} y_L}{y_{22} + y_L}$

solve for $y_L$: $\quad y_i y_{22} + y_i y_L = \Delta_y + y_{11} y_L$

$\Rightarrow (y_i - y_{11}) y_L = \Delta_y - y_{22} y_i$

$y_L = \dfrac{\Delta_y - y_{22} y_i}{y_i - y_{11}}$

Choose $Y(s)$ by




$\Rightarrow Y(s) = Y_c + Y_g = \begin{bmatrix} sC & -sC+g \\ -sC-g & sC \end{bmatrix}$

$Y_c = \begin{bmatrix} sC & -sC \\ -sC & sC \end{bmatrix}, \; Y_g = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$

$\Delta_y = (sC)(sC) - (-sC+g)(-(sC-g)) = g^2$

$$y_L(\alpha) = \frac{\Delta y - y_{22} y_i'}{y_i - y_{i1}} = \frac{g^2 - \alpha c\, y_i'}{y_i - \alpha c} = \frac{g^2\left(1 - \frac{\alpha c}{g} \cdot y_i'(\alpha)/g\right)}{g\left(\frac{y_i'}{g} - \frac{\alpha c}{g}\right)} = g\left(\frac{1 - \frac{\alpha c}{g} \cdot \frac{y_i'(\alpha)}{g}}{\frac{y_i'(\alpha)}{g} - \frac{\alpha c}{g}}\right) = g\left[\frac{k y_i(k) - \alpha y_i(\alpha)}{k y_i'(\alpha) - \alpha y_i(\alpha)}\right]$$

try $c = 1/k \Rightarrow$

$$f(\alpha) = \frac{k\, z(\alpha) - \alpha\, z(k)}{k\, z(k) - \alpha\, z(\alpha)}$$

$$y_L = \frac{g^2\left(1 - \frac{\alpha c}{g} \cdot \frac{y_i(\alpha)}{g}\right)}{g\left(\frac{y_i}{g} - \frac{\alpha c}{g}\right)}$$

$$\boxed{g = y_i(k)}, \quad y_L(\alpha) = y_i(k)\left[\frac{1 - \frac{\alpha c}{y_i(k)} \cdot \frac{y_i(\alpha)}{y_i(k)}}{\frac{y_i(\alpha)}{y_i(k)} - \frac{\alpha c}{y_i(k)}}\right]$$

$$\Rightarrow y_L = y_i(k)\left\{\frac{y_i(\alpha) - \frac{\alpha c}{y_i(k)} \cdot y_i(\alpha)}{y_i(\alpha) - \alpha c}\right\}$$

let $\frac{c}{y_i(k)} = \frac{1}{k} \Rightarrow \boxed{c = \frac{y_i(k)}{k}}$

$$= y_i(k)\left[\frac{y_i(k) - \frac{1}{k}\alpha \cdot y_i(\alpha)}{y_i(\alpha) - \alpha \cdot \frac{y_i(k)}{k}}\right]$$

$$y_L(\alpha) = y_i(k)\left[\frac{k\, y_i(k) - \alpha\, y_i(\alpha)}{k\, y_i(\alpha) - \alpha \cdot y_i(k)}\right]$$

$\Leftarrow$ a Richards' function is PR if $y_i(\alpha)$ is PR & $k$ is $k > 0$ (real & positive)

here $k-\alpha$ factors numerator & denominator

so $\delta[y_L(\alpha)] = \delta[y_i(\alpha)]$

but can choose $k$ such that $k+\alpha$ also cancels $\Rightarrow \delta[y_L(\alpha)] = \delta[y_i(\alpha)] - 1$

to choose $k$: choose $\left.\frac{y_i(k) + y_i(-k)}{2} = Ev\, y(s)\right|_{s=k} = 0$

if $y_i(s)$ is LPR then $Ev[y_i(s)] \equiv 0$ for all $s$ $\Rightarrow$ if LPR can choose any real $k > 0$

$$y_L(s) = y_{i_1}(s)\left[\frac{k y_i(k) - s y_i(s)}{k y_i(s) - s y_i(k)}\right] \qquad y_i(-k) = -y_i(k) \text{ @ a zero of } Ev\, y_i(s)$$

$\therefore$ if $k$ is a zero of $Ev[y_i(s)]$ then if $y_i(s)$ is rational then $(s-k)(s+k)$ both cancel

& $\delta[y_L(s)] = \delta[y_i(s)] - 1$

$\Rightarrow y_L$ is simpler &

repeat until $\delta[y_{...}(s)] = 0$ (a resistor $\Rightarrow$ open if $y = 0$)

Ex: $y(s) = \frac{2s(s^2+2)}{(s^2+1)}$ $\quad k$: let $k = 1$ $\quad Ev[y(s)] = \frac{1}{2}\left[y(s) + y(-s)\right] = \frac{1}{2}\left[\frac{2s(s^2+2)}{(s^2+1)} + \frac{-2s(s^2+2)}{(s^2+1)}\right] = 0$

$y(k) = y(1) = \frac{2(3)}{2} = 3 \qquad g = y(k) = 3, \quad c = \frac{y(k)}{k} = \frac{3}{1}$

$$y_L(s) = 3\left[\frac{1 \cdot 3 - s\left(\frac{2s(s^2+2)}{s^2+1}\right)}{1 \cdot s\left(\frac{2s(s^2+2)}{s^2+1}\right) - s \cdot 3}\right] = 3\left[\frac{3(s^2+1) - 2s^2(s^2+2)}{2s^2(s^2+2) - 3s(s^2+1)}\right] = \frac{(s+1)(s-1)}{(s+1)(s-1)} \cdot \ldots$$