ENEE 610
10/03/17

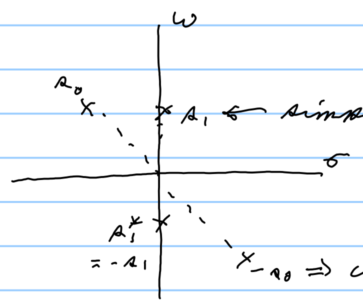
Note Title 10/2/2017

lossless 1-port $\quad y(s) + y(-s) = 0 \Rightarrow y(s) = -y(-s) \Rightarrow$ odd in s

$\text{Ev } y(s) = \frac{1}{2}\left(y(s) + y(-s)\right)$

$\text{od } y(s) = \frac{1}{2}\left(y(s) - y(-s)\right) \qquad y(s) = \text{Ev } y + \text{od } y$

$s = \sigma + j\omega$

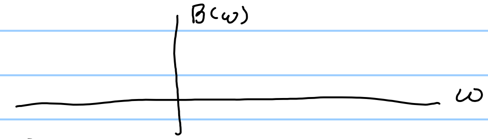


$\leftarrow$ simple for PR function with residue $k > 0$

$\Rightarrow$ can't be a pole for a PR $y(s) \Rightarrow s_0$ can't be for lossless $y(s) \Rightarrow$ all poles on ω axis

lossless $\quad y(s) = \sum_{i=1}^{m} \frac{2k_i s}{s^2 + \omega_i^2} + k_\infty s \qquad k_i > 0 \quad i = 1, \ldots, m, \infty$

$y(j\omega) = j B(\omega) \qquad$ look @ $\frac{dB(\omega)}{d\omega}$



$\frac{dB(\omega)}{d\omega}, \qquad B(\omega) = \sum_{i=1}^{m} \frac{2k_i \omega}{\omega_i^2 - \omega^2} + k_\infty \omega$

$$\frac{dB(\omega)}{d\omega} = k_\infty + \sum_{i=1}^{m}\left[\frac{2k_i}{\omega_i^2 - \omega^2} + (2k_i\omega)\frac{-1}{(\omega_i^2 - \omega^2)^2}\cdot(-2\omega)\right] = k_\infty + \sum_{i=1}^{m} \frac{2k_i}{(\omega_i^2 - \omega^2)^2}\left[\omega_i^2 - \omega^2 + 2\omega^2\right]$$

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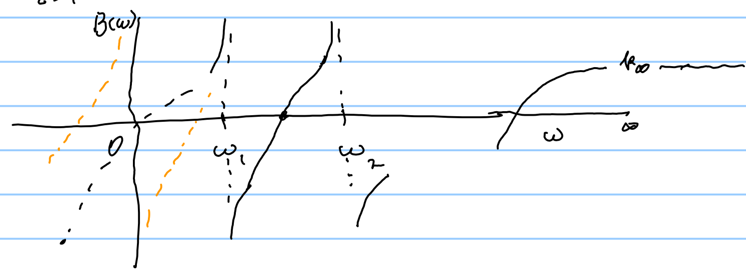
$$\frac{dB(\omega)}{d\omega} = k_\infty + \sum_{i=1}^{m} \frac{2k_i}{(\omega_i^2-\omega^2)^2}(\omega_i^2+\omega^2) \geq 0 \quad \& \ \infty \text{ at } \omega = \omega_i \Leftrightarrow \text{pole points on } \omega \text{ axis}$$



shows zeros & poles alternate for LPR function

$y(s)$ = odd & poles & zeros alternate with $y(1) \geq 0 \Longleftrightarrow$ LPR

$$= \frac{s\,N_1(s^2)}{D_1(s^2)} \text{ or } \frac{1}{s}\frac{N_2(s^2)}{D_2(s^2)}$$

$$= \frac{N(s)}{D(s)}$$ can synthesize by 1st & 2nd Foster

$\Downarrow$ partial fraction expansion of $z(s) = y(s)$

$\Downarrow$ partial fraction expansion of $y(s)$

Cauer synthesis: Continued fraction expansions (1st about ∞, 2nd about 0)

$$y(s) = k_\infty s + \sum_{k=1}^{m} \frac{2k_i s}{s^2+\omega_i^2} = k_\infty s + \left[\cfrac{1}{z_2(s) = \cfrac{1}{\sum \frac{2k_i s}{s^2+\omega_i^2}}}\right] \leftarrow \text{has a pole @ } \infty$$

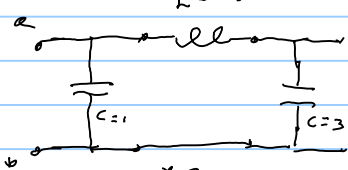
Ex: $y(s) = \dfrac{s(s^2+2)}{(s^2+1/2)} = k_\infty s + \dfrac{2ks}{s^2+1/2} = s + \cfrac{1}{\underbrace{\frac{2}{3}s + \cfrac{1}{3s}}_{\propto\, z(s)}}$ = continued fraction expansion of $y(s)$ about $s = \infty$

LPR $= \dfrac{s^3+2s}{s^2+1/2}$

$$\begin{array}{r|l} & s \\ \hline s^2+1/2 & s^3+2s \\ & s^3+\frac{1}{2}s \\ \hline & (2-\frac{1}{2})s \\ & s^2+1/2 \end{array} \Rightarrow \begin{array}{r|l} & \frac{2}{3}s \\ \hline \frac{3}{2}s & s^2+1/2 \\ & s^2 \\ \hline & 1/2 \end{array} \quad \begin{array}{r|l} & 3s \\ \hline 1/2 & \frac{3}{2}s \end{array}$$

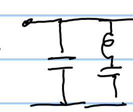
$\delta[y(s)]$ = degree $y(s) = 3$

$y = \dfrac{s(s^2+2)}{s^2+1/2}$ $\longrightarrow$

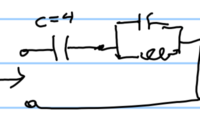


1st Cauer

2nd Foster $y(s) = s + \dfrac{3/2\, s}{s^2+1/2} \Rightarrow$



1st Foster $z(s) = \dfrac{s^2+1/2}{s(s^2+2)} = \dfrac{1/4}{s} + \dfrac{2\hat{k}s}{s^2+2}$



same δ numbers of L's & C's

2nd Cauer = continued fraction expansion about $s = 0$

$y(s) = \dfrac{s(s^2+2)}{(s^2+1/2)} = \dfrac{1}{z(s)}$, $\quad z(s) = \dfrac{s^2+1/2}{s(s^2+2)} = \dfrac{1/2+s^2}{2s+s^3}$

$$z(s) = \frac{\frac{1}{2} + s^2}{2s + s^3}$$

$$= \frac{1}{4s} + \cfrac{1}{\frac{8}{3s} + \cfrac{1}{\frac{3}{4s} + 0}}$$

$z_1 \rightarrow \frac{1}{4s}$, $y_2 \rightarrow \frac{8}{3s}$, $z_3 \rightarrow \frac{3}{4s}$

continued fraction expansion about $s = 0$

$2s + s^3 \,\big)\, \frac{1}{2} + s^2 \quad$ quotient $\frac{1}{4s}$

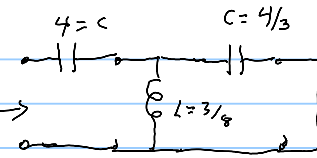
$\frac{1}{2} + \frac{1}{4}s^2$

$(1 - \frac{1}{4})s^2 = \frac{3}{4}s^2 \,\big)\, 2s + s^3 \quad$ quotient $\frac{8}{3s}$

$2s$

$s^3 \,\big)\, \frac{3}{4}s^2 \quad$ quotient $\frac{3}{4s}$

$y = \frac{1}{z} \rightarrow$

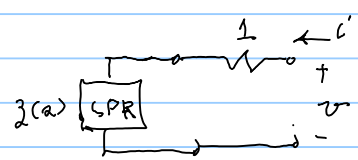


2 nd Cauer for the given $y(s) = \frac{2s + s^3}{1/s + s^2}$

1st Cauer $\Rightarrow$ ladder circuits with 0's of transmission @ $s = \infty$ $\Rightarrow$ low-pass
2nd Cauer $\Rightarrow$ " " " " of " " @ $s = 0$ $\Rightarrow$ high-pass

with 1st & 2nd Foster give minimal realizations using $\delta[y] = \delta[z]$, L's & C's

Hurwitz test: strictly Hurwitz polynomial $\Leftrightarrow$ no zeroes in $\sigma \geq 0$
Hurwitz $\Leftrightarrow$ no zeroes in $\sigma > 0$, on $\sigma = 0$ only simple zeroes



$v = (z + 1)\, i$

$= (\frac{n}{d} + 1)\, i = (\frac{n + d}{d})\, i$

$z = \frac{n(s)}{d(s)}$

given $P(s) = n(s) + d(s)$

form, $z(s) = \frac{n}{d}$

$P(s) = 6s^4+5s^3+4s^2+3s+2 = (6s^4+4s^2+2)+(5s^3+3s)$

$Z(s) = \dfrac{5s^3+3s}{6s^4+4s^2+2}$ is this LPR? if so $P(s)$ is Hurwitz

$Z(s) \Rightarrow$

$5s^3+3s \,\overline{)\,6s^4+4s^2+2}$ quotient $\frac{6}{5}s$

$6s^4+\frac{18}{5}s^2$

$\frac{2}{5}s^2+2 \,\overline{)\,5s^3+3s}$ quotient $\frac{25}{2}s$

$5s^3+25s$

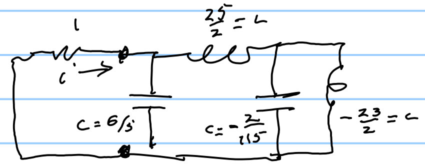
$-23s \,\overline{)\,\frac{2}{5}s^2+2}$ quotient $\frac{-2}{5 \times 23}s$

$\frac{2}{5}s^2$

$2 \,\overline{)\,-23s}$ quotient $-\frac{23}{2}s$

$-23s$

as negative (capacitor) so $P(s)$ not Hurwitz



unstable = $i \rightarrow \infty$ when use initial condition