ENEE 610
09/28/17

Note Title 9/23/2017

$\mathrm{Re}\, V^{T*} I = \frac{1}{2}\left(V^{T*} I + V^{T} I^{*}\right) = \frac{1}{2}\left(V^{T*} I + I^{T*} V\right)$, $I = Y(s) V$

$= \frac{1}{2}\left(V^{T*}[Y(s)V] + [V^{T*} Y^{T*}(s)]V\right)$ if Y is $n \times n$, V is an n-vector

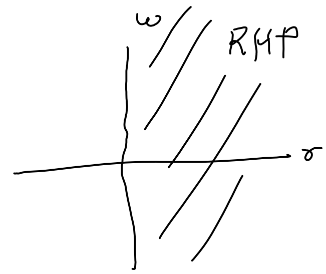
$= V^{T*}\left[\frac{Y(s) + Y^{T*}(s)}{2}\right]V \geq 0$ on $s = j\omega$, $\Rightarrow$ passive

$Y^{H}(s) = \frac{1}{2}\left(Y(s) + Y^{T*}(s)\right)$ is positive semidefinite for passive circuits on $s = j\omega$

If passive no poles in $\sigma > 0$ if rational (as stable)

$Y^{*}(s) = Y(s^{*})$

Look at $e^{-V^{T*}Y(s)V}$ is analytic in $\sigma > 0$



by the maximum modulus theorem

$\left|e^{-V^{T*}Y(s)V}\right|$ is maximum on $\sigma = 0$

$\left|e^{-\mathrm{Re}\,V^{T*}YV - j\,\mathrm{Im}\,V^{T*}YV}\right| = e^{-\mathrm{Re}\,V^{T*}Y(s)V}$ is max of RHP on $\sigma = 0$

$\Rightarrow \mathrm{Re}\,V^{T*}Y(s)V \geq 0$ in $\sigma > 0$ $\Rightarrow \frac{Y(s) + Y^{T*}(s)}{2}$ is positive semidefinite in $\sigma > 0$

Positive real conditions are

1) $Y^{*}(s) = Y(s^{*})$ for $\sigma > 0$ real circuit

2) $Y(s)$ is analytic for $\sigma > 0$ stable

3) $Y(s) + Y^{T*}(s)$ is positive semi-definite in $\sigma > 0$; passive

if rational then call BR $\Longleftrightarrow$ can build a finite passive RLC gyrator circuit

$\Rightarrow$ BR = physically realizable

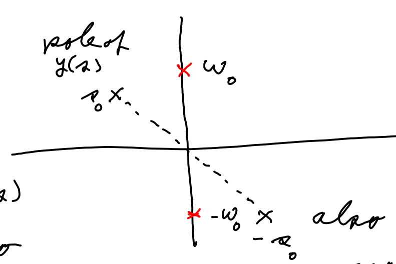
Lossless if $Y(s) + Y^{T}(-s) = 0_n \Rightarrow Y(s) = -Y^{T}(-s)$

for $n = 1$, $y(s) = -y(-s)$

$\therefore$ for lossless $y(s)$, BR, all poles are on $j\omega$ axis

also holds for LBR = lossless BR, $z(s)$

$\Rightarrow$ implies zeroes of $y(s)$ are also on $j\omega$ axis



also a pole of $y(s)$ not allowed if BR

near a pole $y(s) \approx \frac{k}{(s - j\omega_0)^n} + \cdots = \frac{k}{R^n (e^{j\theta})^n}$, $k = |k| e^{j\angle k}$

$y(s) = \left(\frac{|k|}{R^n}\right) e^{j(\angle k - \theta n)}$, $-\pi/2 < \theta < \pi/2$

will mean $\angle k = 0$, $n = 1$

$\mathrm{Re}\, y(s)\Big|_{\text{on semicircle}} = \left(\frac{|k|}{R^n}\right)\cos(\angle k - \theta n) \Rightarrow \angle k = 0$, $n = 1$

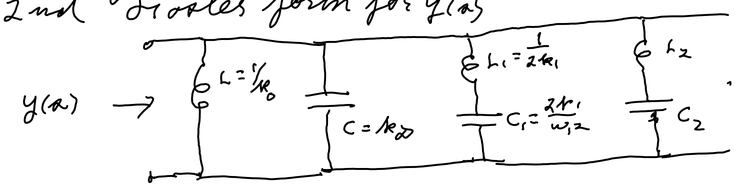
$\Rightarrow$ any finite pole is simple ($r=1$) & with real positive residue k

$$y(s) = \sum_{\substack{\text{poles}\\ \omega_j}} \frac{k_j}{s-\omega_j} + k_\infty s \,, \quad k_j > 0 \,,\ k_\infty > 0$$

$$= \frac{k_0}{s} + k_\infty s + \underbrace{\frac{k_1}{s-j\omega_1} + \frac{k_1}{s+j\omega_1}}_{\frac{2k_1 s}{s^2+\omega_1^2}} + \frac{k_2}{s-j\omega_2} + \frac{k_2}{s+j\omega_2} + \cdots$$

$$\frac{2k_1 s}{s^2+\omega_1^2} + \frac{2k_2 s}{s^2+\omega_2^2} + \cdots$$

2nd Foster form for $y(s)$



$$y(s) = \frac{k_0}{s} + k_\infty s + \sum_{j=1}^{m} \frac{2k_m s}{s^2+\omega_m^2}$$

$$= \frac{N(s)}{D(s)} \,, \quad y(-s) = -y(s)$$

$$= -\frac{N(-s)}{D(-s)}$$

$$y_1(s) = \frac{2k_1 s}{s^2+\omega_1^2}$$

$$z_1(s) = \frac{s^2+\omega_1^2}{2k_1 s} = \frac{s}{2k_1} + \frac{1}{\frac{2k_1}{\omega_1^2} s}$$

$$1/L_1C_1 = \frac{2k_1}{\frac{2k_1}{\omega_1^2}} = \omega_1^2 \Rightarrow \omega_1 = \frac{1}{\sqrt{L_1 C_1}}$$

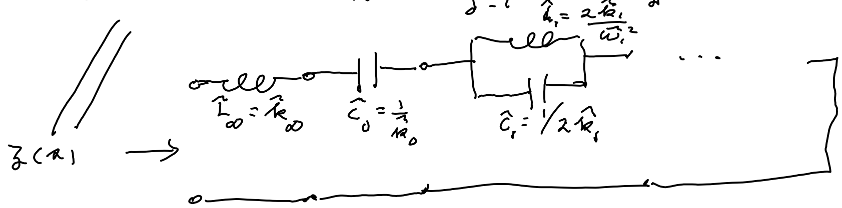
Ex: $$y(s) = \frac{3}{s} + 2s + \frac{6s}{s^2+4} = \frac{3(s^2+4) + 2s(s)(s^2+4) + (s)(6s)}{s(s^2+4)} = \frac{2s^4 + 13s^2 + 12}{s(s^2+4)}$$

$$= \frac{\text{Even in } s}{\text{odd in } s} = \frac{\text{Even in } s}{s \times \text{Even in } s}$$

as $z(s)$ is LBR if $y(s)$ is, can use same idea

$$z(s) = \hat{k}_\infty s + \frac{\hat{k}_0}{s} + \sum_{j=1}^{n} \frac{2\hat{k}_j s}{s^2+\hat{\omega}_j^2}$$

$$z_j = \frac{2\hat{k}_j s}{s^2+\hat{\omega}_j^2} \Rightarrow y_j = \frac{s^2+\hat{\omega}_j^2}{2\hat{k}_j s} = \frac{s}{2\hat{k}_j} + \frac{1}{\frac{2\hat{k}_j}{\hat{\omega}_j^2} s}$$



1st Foster

Ex: $$y(s) = \frac{2s^4 + 13s^2 + 12}{s(s^2+4)} = \frac{3}{s} + 2s + \frac{2k_1 s}{s^2+4}$$

$$= \frac{2\left[\left(s^2 + \frac{13}{4} + \frac{1}{4}\sqrt{73}\right)\left(s^2 + \frac{13}{4} - \frac{1}{4}\sqrt{73}\right)\right]}{s(s^2+4)}$$

$$s^4 + \frac{13}{2}s^2 + 6 \Rightarrow s^2_{1,2} = -\frac{13}{4} \pm \frac{1}{2}\sqrt{\left(\frac{13}{2}\right)^2 - 4 \times 6}$$

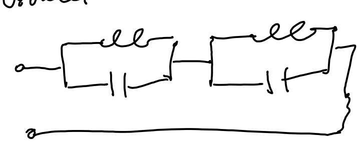
$$= -\frac{13}{4} \pm \frac{1}{4}\sqrt{13^2 - 16 \times 6} \quad (13^2 = 169 - 96)$$

$$= -\frac{13}{4} \pm \frac{1}{4}\sqrt{73}$$

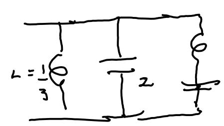
13 × 13 = 39 + 130 = 169; 16 × 6 = 96; 169 − 96 = 73

$$z(s) = \frac{s(s^2+4)}{2\left(s^2 + \frac{13}{4} + \frac{1}{4}\sqrt{73}\right)\left(s^2 + \frac{13}{4} - \frac{1}{4}\sqrt{73}\right)} = \frac{k_1 s}{s^2 + \frac{13}{4} + \frac{1}{4}\sqrt{73}} + \frac{k_2 s}{s^2 + \frac{13}{4} - \frac{1}{4}\sqrt{73}}$$

1st Foster



2nd Foster



these differ but have the same input y & $z = 1/y$

use the minimum number of L's & C's = 4 total = degree of $y(s)$ = degree of $z(s)$ = highest power of s in y & z.