Note Title 9/20/2017

ENEE 610
09/21/17

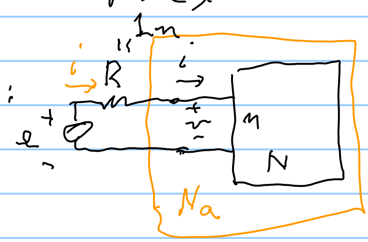
Degree: $T(s) = \dfrac{s^6 + 3s^5 + s^4 \cdots + 10}{s^4 + 5s^3 \cdots + 11}$, $\delta[T(s)] = 6$

$\delta[T(s)]$ = highest power of s in $T(s)$ [a scalar transfer function]

$E\dot{x} = Ax + Bu$
$y = Cx$

$T(s) = C[Es - A]^{-1}B$, $\delta[T(s)] = \text{rank } E$ = McMillan degree

Passive:

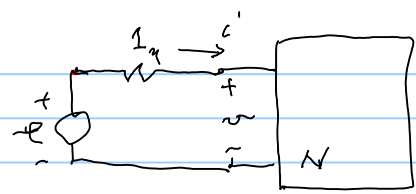


$\Rightarrow i = Y_a \cdot e$
$= Y_a(v + i)$ $\Longrightarrow$ $Y_a v = (1_n - Y_a)\, i$ $\Rightarrow$ $Av = Bi$
$\Longrightarrow A \cdot v = B \cdot i$

$e = v + i$; $v = v^i + v^r$
$i = v^i - v^r$

$A(v^i + v^r) = B(v^i - v^r)$
$(A - B)v^i = -(B + A)v^r \Rightarrow v^r = S \cdot v^i$
$(B + A)^{-1}(B - A)v^i = v^r$

$S = (B + A)^{-1}(B - A)$
$= (1_n - Y_a + Y_a)^{-1}(1_n - Y_a - Y_a)$
$= 1_n^{-1}(1_n - 2Y_a)$ $\Longrightarrow$ $S = 1_n - 2Y_a$. Y_a = admittance matrix for N_a



if $\int_{-\infty}^{\infty} e(t)^T \cdot e(t)\,dt \geq 0$ & finite then $e(t) \in \mathcal{L}^2$

$= \int_{-\infty}^{\infty} (v+i)^T (v+i)\,dt = \int_{-\infty}^{\infty} [v^T v + i^T i + \underbrace{2 v^T i}_{\text{passive forces}}]\,dt$

$\int_{-\infty}^{\infty} v^T i\,dt \geq 0$

if N is passive v & i are in $\mathcal{L}^2$ if e is in $\mathcal{L}^2$

$\Rightarrow$ $v^i = \frac{v+i}{2}$, $v^r = \frac{v-i}{2}$ as sums of v, i in $\mathcal{L}^2$

both v^i & v^r are in $\mathcal{L}^2$

$v = v^i + v^r$, $i = v^i - v^r$; $v^T i = (v^i + v^r)^T (v^i - v^r) = v^{iT} \cdot v^i - v^{rT} v^r$

$(+ v^{rT} v^i - \underbrace{v^{iT} v^r}_{v^{rT} \cdot v^i}) = 0$

$\therefore \int_{-\infty}^{\infty} [v^{iT}_{(t)} \cdot v^i_{(t)} - v^{rT}_{(t)} v^r_{(t)}]\,dt \geq 0$ if $e \in \mathcal{L}^2$ & N is passive; $x(t) \in \mathcal{L}^2$

if $\int_{-\infty}^{\infty} x^T_{(t)} x(t)\,dt$ exists

$= \int_{-\infty}^{\infty} [V^{iT*}(j\omega) V^i(j\omega) - V^{rT*}(j\omega) V^r(j\omega)] \frac{d\omega}{2\pi}$

$x^T x = [x_1, x_2, x_3 \ldots] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \end{bmatrix} = \sum_{k=1}^{n} x_k^2$

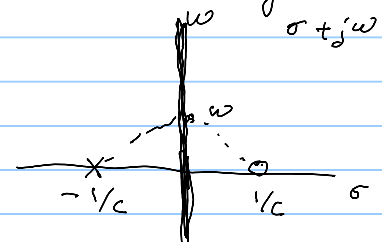
by Parseval's theorem, $^* =$ complex conjugate

$= \int_{-\infty}^{\infty} [V^{iT*}(j\omega) V^i(j\omega) - V^{iT*}(j\omega) S^{T*}(j\omega) S(j\omega) \cdot V^i(j\omega)] \frac{d\omega}{\pi}$

$$= \int_{-\infty}^{\infty} V^{iT*}(j\omega)\left[1_n - S^{T*}(j\omega)\, S(j\omega)\right] V^{i}(j\omega)\, \frac{d\omega}{\pi} \geq 0 \quad \text{holds for all } V^{i}(j\omega) \in \mathcal{L}_2$$

$\therefore$ the Hermitian form $1_n - S^{T*}(j\omega) S(j\omega)$ is positive semidefinite, True for almost all ω

$\sigma + j\omega \equiv s$



note if $f(s) = \frac{n(s)}{d(s)} = \frac{s^n + \dots + a_0}{s^m + \dots + b_0} k_0$ comes from a real circuit, all coefficients are real

$$f^*(s) = \frac{s^{*n} + \dots a_0}{s^{*m} \dots b_0} k_0$$

$$1_n - S^{T}(-j\omega) S(j\omega) \Big|_{s = j\omega} \Rightarrow 1_n - S^{T}(-s) S(s)$$

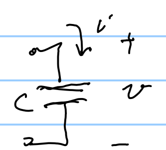
same on $j\omega$ axis but now in whole s-plane

Ex: $y(s) = \frac{3s}{s^2+1}$; $y^*(s) = \frac{3s^*}{(s^*)^2+1}$

only conjugates s

here $s = j\omega$ in $S(j\omega)$; $S^*(j\omega) = S(-j\omega)$

Simple example:



$\Rightarrow y(s) = sc \Rightarrow S = (1_n + Y(s))^{-1}(1_n - Y(s))$

$$S = \frac{1 - sc}{1 + sc} = S(s) \;;\quad S(j\omega) = \frac{1 - j\omega c}{1 + j\omega c}$$

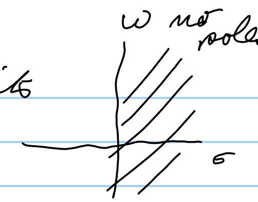
$$1 - S^{*}(j\omega) S(j\omega) = 1 - \left(\frac{1 + j\omega c}{1 - j\omega c}\right)\left(\frac{1 - j\omega c}{1 + j\omega c}\right) \equiv 0$$

pole @ $sc = -1$, $s = -1/c$
zero @ $sc = 1$, $s = 1/c$

$\therefore$ for a passive N, $S(s)$ exists, if rational has real coefficients

no poles on $s = j\omega$ or in $\sigma \geq 0$

& $1_n - S^{T*}(j\omega) S(j\omega)$ is positive semidefinite



$\Rightarrow$ $S(s)$ is bounded-real.

def: a) $S^*(s) = S(s^*)$ real components

b) $S(s)$ is analytic in $\sigma > 0$ & if rational also on $j\omega$ axis stable circuit

c) $1_n - S^{T*}(j\omega) S(j\omega)$ is positive semi definite for almost all real ω

$\nexists$ positive

Lossless: $\mathcal{E}(\infty) = 0 \Rightarrow \displaystyle\int_{-\infty}^{\infty} V^{i\,T*}\left(1_n - S^{T*}(j\omega) S(j\omega)\right) V^i \frac{d\omega}{2\pi} = 0$ for all $V^i \in \mathcal{L}^2$

$\Rightarrow 1_n - S^{T*}(j\omega) S(j\omega) = 0_n \Rightarrow 1_n = S^{T*}(j\omega) S(j\omega) \Rightarrow 1_n = S^T(-s) S(s)$

$\left(S^T(-s)\right)^{-1} = S(s) \Rightarrow S^{-1}(s) = S^T(-s) \Leftarrow$ lossless

Hermitian conjugate

Ex:

$S(s) = \dfrac{1 - sC}{1 + sC}$, $S(-s) = \dfrac{1 - (-s)C}{1 + (-s)C} = \dfrac{1 + sC}{1 - sC} = \dfrac{1}{S(s)}$

$\Rightarrow$ go to $\underbrace{Y(s) \text{ \& } Z(s)}_{\text{positive real}} \Leftrightarrow$ passive