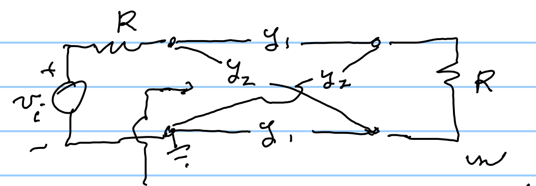
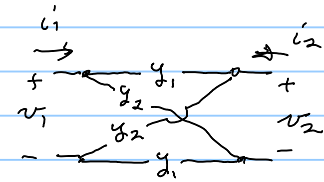
ENEE 610
09/19/17

Note Title 9/19/2017

Constant R lattices

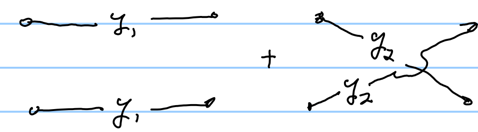




$i = Yv = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Y \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$Y = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$

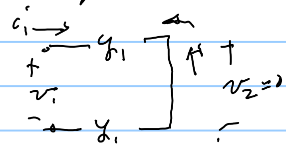
$\Downarrow$



$\Downarrow$

$Y_1 = \begin{bmatrix} \frac{y_1}{2} & -\frac{y_1}{2} \\ -\frac{y_1}{2} & \frac{y_1}{2} \end{bmatrix}$ $\quad Y_2 = \begin{bmatrix} \frac{y_2}{2} & +\frac{y_2}{2} \\ \frac{y_2}{2} & \frac{y_2}{2} \end{bmatrix}$

$y_{11} = \left.\frac{v_1}{i_1}\right|_{v_2=0}$



$= \frac{1}{2} y_1$

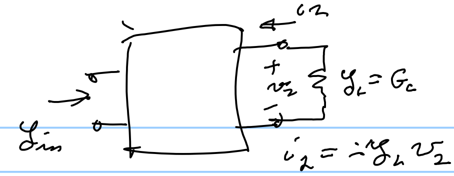
$y_{12} = \left.\frac{i_1}{v_2}\right|_{v_1=0}$

$= y_{21} = \left.\frac{i_2}{v_1}\right|_{v_2=0} = -\frac{y_1}{2}$

$y_{22} = y_{11}$ by symmetry

$$Y = Y_1 + Y_2 = \frac{1}{2} \begin{bmatrix} y_1 + y_2 & y_2 - y_1 \\ y_2 - y_1 & y_2 + y_1 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ -y_L v_2 \end{bmatrix}$$



$$-y_L v_2 = y_{21} v_1 + y_{22} v_2 \Rightarrow -(y_L + y_{22}) v_2 = y_{21} v_1$$

$$\Rightarrow \frac{v_2}{v_1} = -\frac{y_{21}}{y_L + y_{22}}$$

$$i_1 = y_{11} v_1 + y_{12} v_2 = y_{11} v_1 + y_{12} \cdot \frac{(-y_{21})}{y_L + y_{22}} \cdot v_1 = \left[\frac{y_{11} y_{22} + y_L y_{11} - y_{12} y_{21}}{y_L + y_{22}} \right] \cdot v_1$$

$$y_{in} = \frac{\det Y + y_L y_{11}}{y_L + y_{22}} \Rightarrow \text{constant } R \Rightarrow y_{in} = G_L,\ y_L = G_L$$

$$G_L^2 + G_L y_{22} = \det Y + G_L y_{11} \qquad \text{but } y_{22} = y_{11} \text{ for the above symmetric lattice}$$

$$\Rightarrow G_L^2 = \det Y$$

$$= y_{11} y_{22} - y_{21} y_{12} = y_{11}^2 - y_{12}^2$$

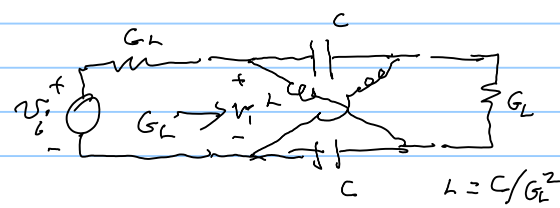
$$= \frac{1}{4}(y_1 + y_2)^2 - \frac{1}{4}(y_2 - y_1)^2 = \frac{1}{4}\left[(\cancel{y_1^2} + 2 y_1 y_2 + \cancel{y_2^2}) - (\cancel{y_2^2} - 2 y_1 y_2 + \cancel{y_1^2}) \right] = \frac{+4\, y_1 y_2}{4}$$

$$\Rightarrow \text{constant } R,\quad G_L^2 = y_1 y_2 \Rightarrow y_2 = G_L^2 / y_1$$

Ex: $y_1 = sC$ then $y_2 = G_L^2 / sC = \frac{1}{sL}$

$L = C / G_L^2$

Ex:

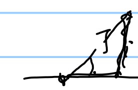


$L = C/G_L^2$

$$v_1 = \frac{1}{2} v_i \quad , \quad v_2 = \frac{-y_{21}}{y_L + y_{22}} \cdot v_1$$

$$y_{21} = \frac{1}{2}(y_2 - y_1), \quad y_{22} = \frac{1}{2}(y_2 + y_1) \Rightarrow \frac{v_2}{v_1} = \frac{-\frac{1}{2}(y_2 - y_1)}{G_L + \frac{1}{2}(y_2 + y_1)} = \frac{y_1 - y_2}{2G_L + (y_2 + y_1)}$$

$$\text{But } y_2 y_1 = G_L^2 \Rightarrow y_2 = \frac{G_L^2}{y_1} \Rightarrow \frac{v_2}{v_1} = \frac{y_1 - G_L^2/y_1}{2G_L + \left(\frac{G_L^2}{y_1} + y_1\right)} = \frac{y_1^2 - G_L^2}{y_1^2 + 2G_L y_1 + G_L^2}$$



$$\frac{v_2}{v_1} = \frac{(y_1 - G_L)(y_1 + G_L)}{(y_1 + G_L)^2} = \frac{y_1 - G_L}{y_1 + G_L}$$

if $y_1(j\omega) = jB(\omega)$, $B(\omega)$ real

Then $$\frac{v_2}{v_1}(j\omega) = \frac{-G_L + jB(\omega)}{G_L + jB(\omega)} = \frac{|-G_L + jB(\omega)|\, e^{j\arctan(-G_L/B(\omega))}}{|G_L + jB(\omega)|\, e^{j\arctan(+G_L/B(\omega))}}$$

$$= 1\, e^{-2j\arctan(G_L/B(\omega))}$$

$y_1(s) = sC$

$y_1(j\omega) = j\omega C$

↑ all pass; changes phase

can cascade keeping constant R's then the gains multiply & keeps all pass

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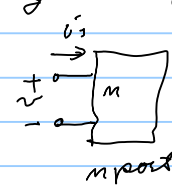
Passivity: $\mathcal{E}(t) \geq 0$ for all time

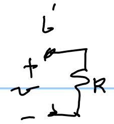
$\mathcal{E}(t)$ = energy into the circuit $= \int_{-\infty}^{t} p(\tau)\, d\tau$

$$= \int_{-\infty}^{t} v^T(\tau)\, i(\tau)\, d\tau \geq 0$$



$p(t) = v^T(t)\, i(t)$

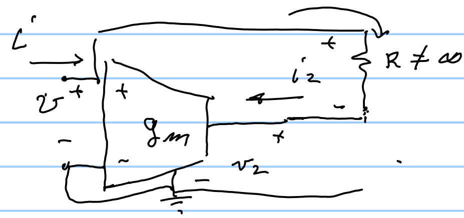
Ex:



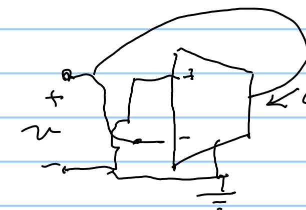
$i = Gv$, $1/R = G$
$G =$ const.

$$\int_{-\infty}^{t} vGv\,d\tau = G\int_{-\infty}^{t} v^2(\tau)\,d\tau = G \times \text{positive if } v \neq 0 \text{ \& real}$$

$\Rightarrow$ passive if $G \geq 0$



$i_2 = g_m v = i$



$\swarrow i_2 = g_m(-v) = -g_m v$

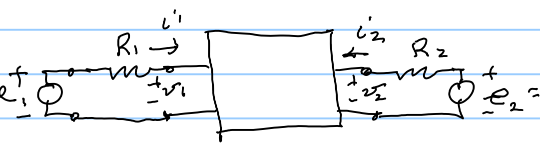
a negative resistor

OTA
operational
transconductance
amplifier
(G component in Spice)

$\Rightarrow$ OTA is not passive as it gives
$G < 0$ $\Leftrightarrow R < 0$ for this connection
$\;\; '' g_m \qquad '' 1/G$

Scattering matrix

$2v_1^i = e_1$



$e_2 = v_2 + i_2 = 2v_2^i \Rightarrow v_2^i = \frac{1}{2} e_2$

$2v^i = v + Ri$ = incident
$2v^r = v - Ri$ = reflected

$R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}$, $R_1 \geq 0$, $R_2 \geq 0$ normalize to $R = 1_2$

$2v^i = v + i$
$2v^r = v - i$
$\Rightarrow$
$v = v^i + v^r$
$i = v^i - v^r$

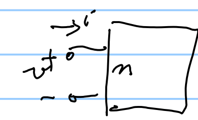
S = scattering matrix : $v^r = S v^i$; $S_{21} = \left.\dfrac{v_2^r}{v_1^i}\right|_{v_2^i = 0}$ $= \dfrac{(v_2 - i_2)/2 = v_2^r}{e_1/2}$

$(\equiv e_2 = 0)$

$S_{11} = \left.\dfrac{v_1^r}{v_1^i}\right|_{v_2^i = 0}$ = reflection coefficient @ port 1 when load in R_2

$= \dfrac{(v_1 - i_1)/2}{(v_1 + i_1)/2} = \dfrac{1 - i_1/v_1}{1 + i_1/v_1} = \left.\dfrac{1 - y_{in}}{1 + y_{in}}\right|$ when load port 2, $R_2 = 1$

$= \dfrac{-v_2}{e_1}$ = voltage gain, loaded in $R_2 = 1$



Before $A(s) v(s) = B(s) i(s)$

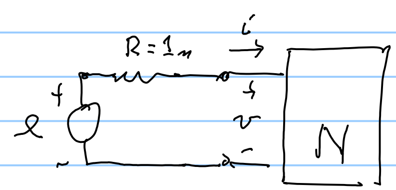
$v = v^i + v^r$
$i = v^i - v^r$ $\Longrightarrow$ $A(v^i + v^r) = B(v^i - v^r)$

$(A - B) v^i = (-A - B) v^r \Longrightarrow (B + A) v^r = (B - A) v^i$, $v^r = S v^i$

$S = (B + A)^{-1} (B - A)$; if $i = Y v \Rightarrow A = Y, B = 1_n$

$= (1_n + Y)^{-1} (1_n - Y)$

note if Y comes from a passive circuit no diagonal entry $= -1$ so $(1_n + Y)^{-1}$ exists



choose e such that $\int_{-\infty}^{t} e^T(\tau) e(\tau)\, d\tau$ is finite ≥ 0

$e = (v + i) \Rightarrow \mathcal{E}(t) = \int_{-\infty}^{t} (v^T v + i^T i + 2 v^T i)\, d\tau \geq 0$, $e \in \mathcal{L}^2[0, t]$

$\therefore$ if N is passive $\int_{-\infty}^{t} v^T i \geq 0$ Then as $\int e^T e \, d\tau \geq 0$ if $e \in \mathcal{L}^2[0,t]$

both $\int v^T v \, d\tau$ & $\int i^T i \, d\tau \geq 0$ & finite $\Rightarrow v, i \in \mathcal{L}^2[0,t]$