Note Title

ENEE 610
09/14/17

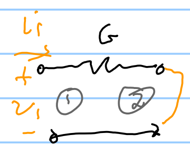
9/14/2017

$$Y(a) = \underset{\text{symmetric}}{Y(s)} + \underset{\text{skew symmetric}}{Y(a)}$$

$$Y_{\text{symmetric}} = \frac{Y(a) + Y(a)^T}{2}, \quad Y_{\text{skew symmetric}} = \frac{Y(a) - Y(a)^T}{2}$$

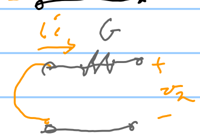
$$Y = \begin{bmatrix} 3 & -2 \\ 8 & 4 \end{bmatrix} \qquad Y_{\text{symmetric}} = \left\{ \begin{bmatrix} 3 & -2 \\ 8 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 8 \\ -2 & 4 \end{bmatrix} \right\} / 2 = \begin{bmatrix} 3 & 3 \\ 3 & 4 \end{bmatrix}$$

$$I = YV \qquad Y_{\text{skew}} = \frac{\begin{bmatrix} 3 & -2 \\ 8 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 8 \\ -2 & 4 \end{bmatrix}}{2} = \underbrace{\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}}_{\text{gyrator}}$$



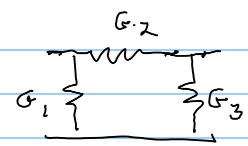
$$\Rightarrow y_{11} = \left.\frac{i_1}{v_1}\right|_{v_2=0} = G$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$



$$y_{12} = \left. i_1 / v_2 \right|_{v_1=0} = -G$$

$$Y = \begin{bmatrix} G & -G \\ -G & G \end{bmatrix}$$

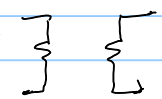


$$Y = \begin{bmatrix} G_1+G_2 & -G_2 \\ -G_2 & G_3+G_2 \end{bmatrix}$$

Can transform $Y = \begin{bmatrix} 3 & 3 \\ 3 & 4 \end{bmatrix}$; $\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 3 & 3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$$

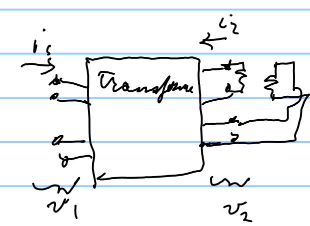
$$\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \Longrightarrow Y = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow$$



But

$$\begin{bmatrix} 3 & 3 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$



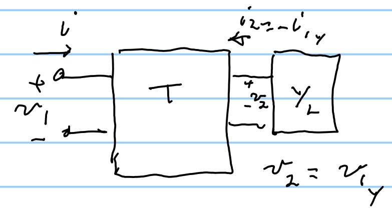
$v_2 = T v_1$

$i_1 = -T^T i_2$

$$P_{in} = 0 = v_1^T i_1 + v_2^T i_2$$
$$= v_1^T i_1 + v_1^T T^T i_2 = v_1^T \left[i_1 + T^T i_2 \right]$$

if holds for all v_1, can cancel v_1

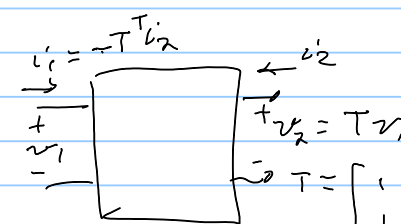
$$0 = i_1 + T^T i_2$$



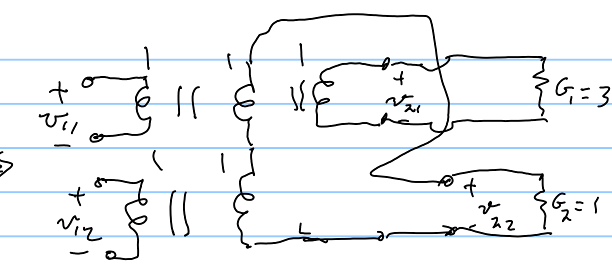
$$i_1 = -T^T(-i_Y) = +T^T(Y_L v_2 \text{ transforms})$$
$$= T^T Y_L T v_1$$

input to this ($4 \Rightarrow 2n$ port) transformer loaded in an ($2 \Rightarrow n$ port) load with admittance Y_L gives

$$Y_{in} = T^T Y_L T$$

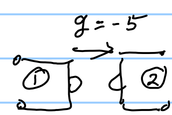


$$T = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

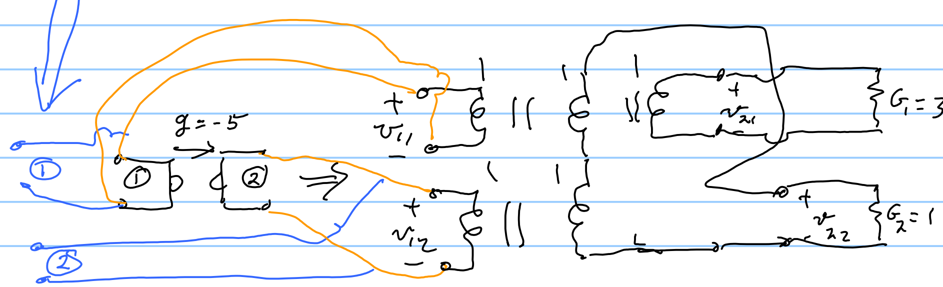
$\Rightarrow$ 

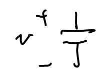
$$Y_{sym} = \begin{bmatrix} 3 & 3 \\ 3 & 4 \end{bmatrix}$$

$$Y_{skew} = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$



$$Y = \begin{bmatrix} 3 & 2 \\ 8 & 4 \end{bmatrix}$$ use a parallel connection





$i = C\frac{dv}{dt}$ then if $v = Ve^{\alpha t}$

$C\frac{dv}{dt} = C\alpha V e^{\alpha t}$

$i = Ie^{\alpha t} = C\alpha V e^{\alpha t}$

$I = C\alpha V$

Back to semi state equations

$$E\dot{x} = \mathcal{A}x + \mathcal{B}u, \quad y = \mathcal{C}_x x \qquad \cdot = \alpha$$

$$PEQ \cdot Q^{-1}\dot{x} = P\mathcal{A}QQ^{-1}x + P\mathcal{B}u, \quad y = \mathcal{C}_x Q Q^{-1} x$$

if LTI then

$$(E\alpha - \mathcal{A})x = \mathcal{B}u \Rightarrow x = (E\alpha - \mathcal{A})^{-1}\mathcal{B}u$$

$$y = \mathcal{C}_x(E\alpha - \mathcal{A})^{-1}\mathcal{B}u$$

But if change semistate from x to $Q^{-1}x = \hat{x}$ a new semistate

then

$$Q^{-1}x = \hat{x} = (PEQ\alpha - P\mathcal{A}Q)^{-1}PBu$$

$$y = C_x Q(PEQ\alpha - P\mathcal{A}Q)^{-1}PBu = C_x QQ^{-1}(E\alpha - \mathcal{A})^{-1}P^{-1}PBu$$

$$= C_x(E\alpha - \mathcal{A})^{-1}Bu$$

Example: If $E = \begin{bmatrix} 4 & 4 \\ -2 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}\begin{bmatrix} 4 & 4 \\ -2 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 0 & 0 \end{bmatrix}$$

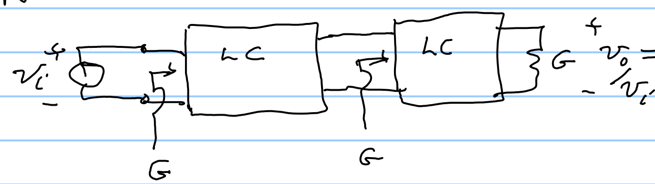
$$\begin{bmatrix} 4 & 4 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1/4 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$PEQ = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \underbrace{\begin{bmatrix} 1/4 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}}_{P}\begin{bmatrix} 4 & 4 \\ -2 & -2 \end{bmatrix}\underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}_{Q} = \begin{bmatrix} 1/4 & 0 \\ 1/2 & 1 \end{bmatrix}\begin{bmatrix} 4 & 4 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

$\therefore$ using elementary matrix transformations we can transform E to $1_k \oplus 0_{b-k}$

k = degree of Es = degree of $C_x(Es - A)^{-1}B$

Constant R



$v_o / v_i \Rightarrow$ all pass function

$$\frac{s-a}{s+a}$$

$$\frac{s^2 - as + b}{s^2 + as + b}$$