Note Title 9/12/2017

EE 610
09/12/17

State equations $\dot{x} = Ax + Bu$
LTI = linear $y = Cx + Du$
time invariant

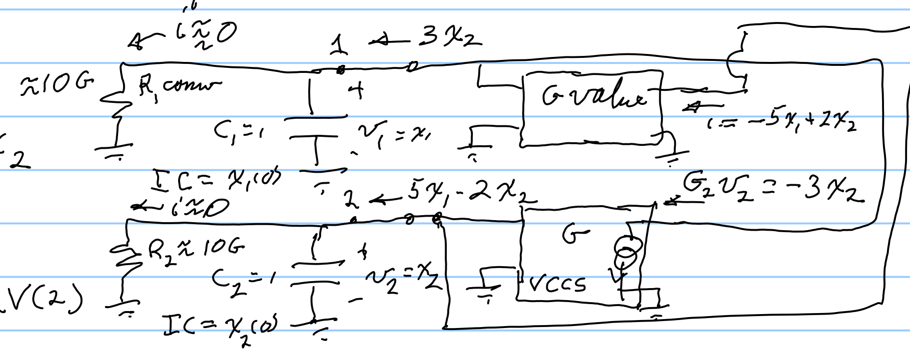
$\dot{x} = dx/dt = sx$
$s = d/dt$
$s = \sigma + j\omega$
$\mathcal{L}[f] = \int_{-\infty}^{\infty} f(t) e^{-st} dt$

capacitor $i = C\, dv/dt$, C = const = capacitance

e^{st} solving ODE

if $C=1$, $\frac{dv}{dt} = \frac{dx}{dt}$ = current, $x \approx$ voltages

$\dot{x}_1 = 3x_2$
$\dot{x}_2 = 5x_1 - 2x_2$
$[x_1(0), x_2(0)]$



G value ⇒ gain $-5V(1) + 2V(2)$

$\dot{x}_1 = 3x_1x_2 \leftarrow$ G value $\quad i = 3V(1)*V(2)$ can't do with G itself as $d(G)$ only allows constants

$\dot{x}_2 = -2x_1$

Semi-state eqs. $\quad \mathcal{E}\dot{x} = \mathcal{A}x + \mathcal{B}u$

LTI $\quad y = \mathcal{C}_x x$

$x = \begin{bmatrix} v_t \\ i_\ell \end{bmatrix}$ = semi-state b-vector

$\mathcal{E}$ is singular

$v_b = \mathcal{C}^T v_t$

$i_b = \mathcal{J}^T i_\ell$

But state-variable

$\dot{x} = Ax + Bu$ $\quad x$ = state

$y = Cx + Du + \underbrace{E\dot{u} + F\ddot{u} + \cdots}$ $\quad \Rightarrow$ k-vector

if ignore $\longrightarrow$

then $\quad s1_k X(s) = AX(s) + BU(s) \Rightarrow (s1_k - A)X = BU$

$Y(s) = CX(s) + DU(s) \quad Y = CX + DU$

$Y = DU + C(s1_k - A)^{-1}BU = [D + C(s1_k - A)^{-1}B] \cdot U$

$$Y(s) = T(s)\cdot U(s) = D + C(sI_n - A)^{-1}B \; ; \text{ as } s\to\infty \Rightarrow T(s)\big|_{s\to\infty} = D$$

can not get $T(s) = s$ unless $E\dot{u} + F\ddot{u} + \dots$

Ex:

$$\begin{bmatrix} s & 0 \\ 0 & 0 \end{bmatrix} x = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$y = \begin{bmatrix} a & b \end{bmatrix} x$$

$$\Rightarrow x_2 = u \;, \quad s x_1 = x_2 = u \Rightarrow y = \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} 1/s\, u \\ u \end{bmatrix} = \frac{a}{s} u + b u$$

$$E\dot{x} = \mathcal{A}x + \mathcal{B}u \qquad \Rightarrow (sE - \mathcal{A})x = \mathcal{B}u \Rightarrow x = (sE - \mathcal{A})^{-1}\mathcal{B}u$$

$$y = \mathcal{C}_x x \qquad\qquad y = \mathcal{C}_x (sE - \mathcal{A})^{-1}\mathcal{B}\cdot u$$

$$T(s) = \mathcal{C}_x (sE - \mathcal{A})^{-1}\mathcal{B}. \qquad \text{assumes } sE - \mathcal{A} \text{ is nonsingular}$$

$$E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \; A = I_2 \Rightarrow \left[\begin{bmatrix} 0 & s \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right]^{-1} = \begin{bmatrix} -1 & s \\ 0 & -1 \end{bmatrix}^{-1} = \frac{1}{1}\begin{bmatrix} -1 & -s \\ 0 & -1 \end{bmatrix}$$

inverse × original = identity $\begin{bmatrix} -1 & -\alpha \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} -1 & \alpha \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} +1 & -\alpha+\alpha=0 \\ 0 & 1 \end{bmatrix} = I_2$

$C_x \begin{bmatrix} -1 & -\alpha \\ 0 & -1 \end{bmatrix} B$ choose $C_x = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & -\alpha \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & -\alpha \end{bmatrix}, \quad \begin{bmatrix} 0 & -\alpha \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \alpha$$

$T(\alpha) = \alpha$ for $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \dot{x} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u$, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

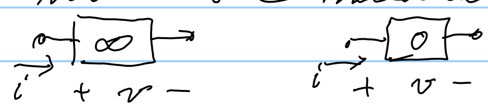
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

$T(\alpha) = \alpha$

∴ the semi-state equations come naturally from circuits & given LTI ones we will be able to create a circuit. State variable equations are essentially a special case.

$$Av = Bi$$

norator & nullator

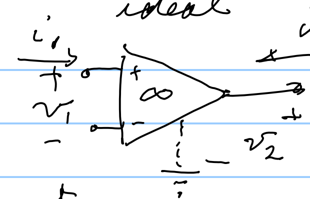


law: v = arbitrary $v=0,\ i=0$

i = arbitrary

(independent of v)

ideal



i_2 can be anything

can be anything

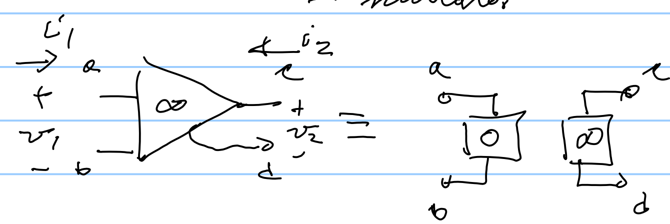
$\Rightarrow$ norator

virtual connection

$v_1 = 0$

$i_1 = 0$

$\Rightarrow$ nullator



desire $Av = Bi$ for an op-amp (ideal)

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

both A & B are singular

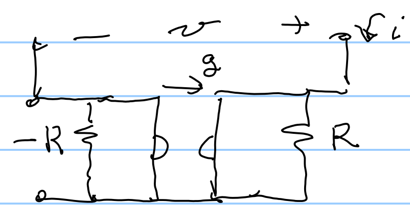
$\Rightarrow$ no 2-port admittance

But norator $\Rightarrow$ $\begin{bmatrix} 0 \end{bmatrix}\begin{bmatrix} v_1 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}\begin{bmatrix} i_1 \end{bmatrix}$

nullator $\Rightarrow$ $\begin{bmatrix} 1 & \cancel{0} \\ 0 & \cancel{0} \end{bmatrix}\begin{bmatrix} v \end{bmatrix} = \begin{bmatrix} 0 & \cancel{0} \\ 1 & \cancel{0} \end{bmatrix}\begin{bmatrix} i \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix} v = \begin{bmatrix} 0 \\ 1 \end{bmatrix} i$

$\Rightarrow$ non square A&B $\Rightarrow$ often hear if have a norator somewhere with its paired a nullator & vice versa.

But



can make either

$$\left.\begin{aligned} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\dot{x} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}x + \begin{bmatrix} 0 \\ -1 \end{bmatrix}u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix}x \end{aligned}\right\}$$

to convert to state variable equations: keep x_2 & eliminate x_1

second eq.: $0 = x_1 - u = \quad x_1 = -u$

1st eq., eliminate x_1: $\dot{x}_2 = x_1 = -u$

let $x = x_2$,

$y = x_1 = -u =$

$$\left.\begin{aligned} \dot{x} &= -u \\ y &= -u \end{aligned}\right\} \text{states variable eqs.}$$

can manipulate by row & column operation

$$PEQ = \begin{bmatrix} I_k & 0 \\ 0 & 0_{n-k} \end{bmatrix};$$

$$E\dot{x} = \mathcal{A}x + \mathcal{B}u$$

$$PEQQ^{-1}\dot{x} = P\mathcal{A}QQ^{-1}x + P\mathcal{B}u$$

$$Q^{-1}x = \hat{x} \Rightarrow \begin{bmatrix} I_k & 0 \\ 0 & 0_{n-k} \end{bmatrix}\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = P\mathcal{A}Q\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + P\mathcal{B}u$$

$\Rightarrow$ solve for $\hat{x}_2 = f(\hat{x}_1)$ to eliminate $\hat{x}_2$