

EE610  
09/05/17

$$u_b = e^T q_r^T, \quad l_b = \sigma^T l_r, \quad e^T = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \quad \sigma^T = \begin{bmatrix} -k \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

$i_b \downarrow \quad \uparrow \quad i_r$

$$v_b^T \cdot l_b = \sum_{k=1}^b v_{b,k} \cdot l_{b,k} \quad v_{b,k} \cdot l_{b,k} = \text{rows into component terms}$$

= Total rows into all branches

$$= v_r^T \cdot e \cdot \sigma^T \cdot l_r, \quad e \cdot \sigma^T = \begin{bmatrix} 1 & -k & \dots & k \end{bmatrix} = 1 \cdot (-k) + k \cdot 1$$

This is zero  $\approx 0_{t \times r}$

$$A \cdot v = \beta \cdot l, \quad v = v_{b-r}, \quad l = l_{b-r}$$

$$\begin{bmatrix} A e^T & -\beta \sigma^T \end{bmatrix} \begin{bmatrix} v_r \\ l_r \end{bmatrix} = A e - \beta j$$

$A = Y, B = I_b \quad A^T = B^T \Rightarrow I_b^T = Y^T$

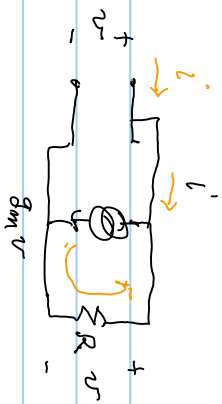
$\Rightarrow \begin{bmatrix} Y e^T & | & -I_b^T \sigma^T \end{bmatrix} x = A e - B^T i \quad \left. \vphantom{\begin{bmatrix} Y e^T & | & -I_b^T \sigma^T \end{bmatrix}} \right\} \text{Norton's equivalent circuit across}$

$x^C \quad \text{Z}_{norton} \quad e \begin{bmatrix} Y e^T & | & -\sigma^T \end{bmatrix} x = \begin{bmatrix} e Y e^T & - e \sigma^T \end{bmatrix} x = e A_2 - e B^T i$   
 $= e Y_{b \times b} e^T v_T^T + (-e \sigma^T i) = e A_2 - e B^T i$

$\underbrace{\quad}_{t \times t} \quad \underbrace{\quad}_{k \times R} \quad \underbrace{\quad}_{t \text{ firing levels}} \quad \underbrace{\quad}_{\text{Norton equivalent}}$

i.e. get  $t$  eqs in  $t$  unknowns,  $v_T$ .  
 if branch by branch gives  $Y_{b \times b}, v = 1, i$ . Then can reduce the number of eqs. from  $b$  to  $t$ .  
 $t + R = b$ ;  $t = m - 1$  (for loopable part)  
 seems like could use the  $t = m - 1$  voltages with reference to ground

Some interesting components



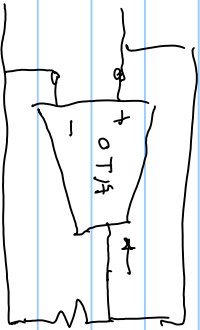
$$i_1 = g_m v + G v, \quad G = 1/R$$

$$= (g_m + G) v$$

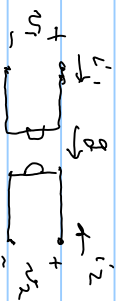
input we see  $y = g_m + G$

as  $g_m$  can be  $< 0$  then  $y$  can be negative

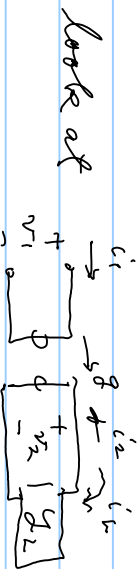
if used an OTA  
 (= operational transconductance  
 amplifiers  $\Rightarrow$  differential pair)



The system



$$\begin{bmatrix} i_1' \\ i_2' \end{bmatrix} = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} ; g = \text{real}$$



$$i_1' = g v_2 ; i_2' = g v_1$$

$$= g \cdot \frac{1}{g} i_1'$$

$$v_1 = \frac{1}{g} \cdot \frac{1}{g} g_L \cdot i_1'$$

$$i_2' = -i_1' = -g v_1 \Rightarrow v_1 = \frac{1}{g} i_2'$$

∥

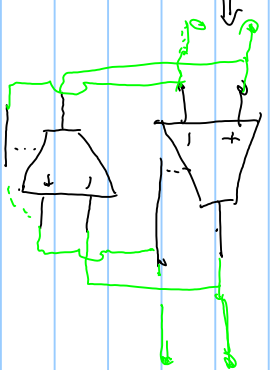
$$C_1 = g_{in} r_1 = \frac{1}{g_L} \times g_m^2 r_1 \Rightarrow g_{in} = \frac{1}{g_L} \cdot g_m^2 \text{ if the load is a capacitor}$$

then  $Y_L(s) = AC$ ,  $g_{in} = \frac{1}{AC} \cdot g_m^2 = \frac{1}{A L_{eq}} \Rightarrow g_{in} = \frac{C}{g_m} R = L_{eq} R$

$C \rightarrow C/g_m = L_{eq}$ . if  $g = \text{gyration conductance} = 10^{-3}$   
 $L_{eq} = 10^6 \cdot C$  as  $C = 1 \mu F \Rightarrow L = 1 \text{ Henry}$

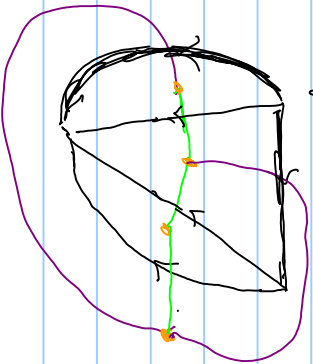
OTA  $\Rightarrow Y_1 = \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix}$

$Y_2 = \begin{bmatrix} 0 & -g_m \\ 0 & 0 \end{bmatrix}$



$Y = Y_1 + Y_2 = \begin{bmatrix} 0 & -g_m \\ g_m & 0 \end{bmatrix}$

Dual of planar graph (can draw with no crossing branches)



• = nodes in dual graph

—  $\Rightarrow$  tree branches in the dual graph

—  $\Rightarrow$  links in dual graph

