

EE610

08/31/17 → 09/01/17

Final preliminary versions of a paper for Stanley

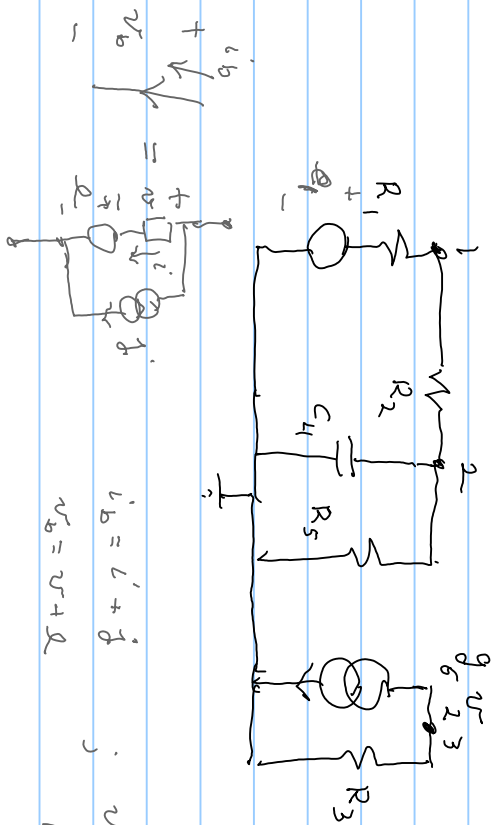
$$Q = \begin{bmatrix} 1 & k \\ 1 & k \end{bmatrix}, \quad \sigma = \begin{bmatrix} -k & 1 \\ 1 & k \end{bmatrix}; \quad Q_t = \begin{bmatrix} 1 & k \\ 1 & k \end{bmatrix} \begin{bmatrix} i_t \\ i_t \end{bmatrix} \Rightarrow i_t = k i_t$$

$$g_t = \begin{bmatrix} -k & 1 \\ 1 & k \end{bmatrix} \begin{bmatrix} v_t \\ v_t \end{bmatrix} = e \cdot i_t$$

$$-k v_t = -v_t \quad i_t = -k i_t = \begin{bmatrix} i_t \\ i_t \end{bmatrix} = \begin{bmatrix} -k \\ 1 \end{bmatrix} i_t = i_t = i_t = \sigma_t^T i_t$$

$$-v_t = -v_t \quad i_t = i_t$$

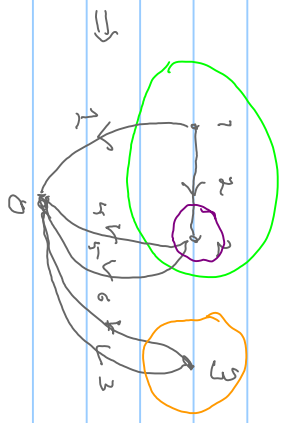
$$\begin{bmatrix} 1 & k \\ k & 1 \end{bmatrix} v_t = \begin{bmatrix} v_t \\ v_t \end{bmatrix} = v_t = e^T v_t$$



$$i_b = i' + j' \quad ; \quad v_b = \mathcal{O}^t v_r'$$

$$v_b = v + \mathcal{R} \quad ; \quad i_b = \mathcal{O}^t i_r'$$

i & j = independent source vectors



$A v = B i$ non independent sources

$$A(v_b - \mathcal{R}) = B(i_b - j')$$

$$A \mathcal{O}^t v_r' - A \mathcal{R} = B \mathcal{O}^t i_r' - B j' \Rightarrow A \mathcal{O}^t v_r' - B \mathcal{O}^t i_r' = A \mathcal{R} - B j'$$

$$\begin{bmatrix} A \mathcal{O}^t & -B \mathcal{O}^t \end{bmatrix} \begin{bmatrix} v_r' \\ i_r' \end{bmatrix} = A \mathcal{R} - B j'$$

; v_r' & i_r' unknown
 unknowns & equations

$$\begin{aligned}
 \mathcal{R} &= \begin{bmatrix} \mathcal{R}_1 \\ \mathcal{R}_2 \\ \mathcal{R}_3 \\ \mathcal{R}_4 \\ \mathcal{R}_5 \\ \mathcal{R}_6 \end{bmatrix}, \quad \mathcal{F} = \begin{bmatrix} \mathcal{F}_1 \\ \mathcal{F}_2 \\ \mathcal{F}_3 \\ \mathcal{F}_4 \\ \mathcal{F}_5 \\ \mathcal{F}_6 \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} \mathcal{B}_1 \\ \mathcal{B}_2 \\ \mathcal{B}_3 \\ \mathcal{B}_4 \\ \mathcal{B}_5 \\ \mathcal{B}_6 \end{bmatrix} \\
 &= \begin{bmatrix} \mathcal{R}_1 & \mathcal{F}_1 & \mathcal{B}_1 \\ \mathcal{R}_2 & \mathcal{F}_2 & \mathcal{B}_2 \\ \mathcal{R}_3 & \mathcal{F}_3 & \mathcal{B}_3 \\ \mathcal{R}_4 & \mathcal{F}_4 & \mathcal{B}_4 \\ \mathcal{R}_5 & \mathcal{F}_5 & \mathcal{B}_5 \\ \mathcal{R}_6 & \mathcal{F}_6 & \mathcal{B}_6 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1.1 \\ 1.2 \\ 1.3 \\ 1.4 \\ 1.5 \\ 1.6 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{A} \mathcal{U} &= \mathcal{B} \mathcal{I} \\
 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} &= \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \mathcal{C} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \\
 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} &= \begin{bmatrix} -1 & +1 & 0 & 1 & 0 & 0 \\ -1 & +1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}
 \end{aligned}$$

$$\begin{aligned}
 &= \begin{bmatrix} \mathcal{R}_1 & \mathcal{F}_1 & \mathcal{B}_1 \\ \mathcal{R}_2 & \mathcal{F}_2 & \mathcal{B}_2 \\ \mathcal{R}_3 & \mathcal{F}_3 & \mathcal{B}_3 \\ \mathcal{R}_4 & \mathcal{F}_4 & \mathcal{B}_4 \\ \mathcal{R}_5 & \mathcal{F}_5 & \mathcal{B}_5 \\ \mathcal{R}_6 & \mathcal{F}_6 & \mathcal{B}_6 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1.1 \\ 1.2 \\ 1.3 \\ 1.4 \\ 1.5 \\ 1.6 \end{bmatrix}
 \end{aligned}$$

Equations & unknowns, $b=6$, $f=3=2$

$$\begin{bmatrix} 1 & 0 & 0 & | & -R_1 & -R_1 & 0 \\ 0 & 1 & 0 & | & R_2 & R_2 & 0 \\ 0 & 0 & 1 & | & 0 & -R_3 & 0 \\ R_4 & -R_4 & 0 & | & 1 & 0 & 0 \\ 1 & -1 & 0 & | & 0 & R_5 & 0 \\ 0 & g_2 & 0 & | & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} 0 & 0 & 0 & | & 0 & 0 & 0 \\ 0 & 0 & 0 & | & 0 & 0 & 0 \\ R_4 & -R_4 & 0 & | & 0 & 0 & 0 \\ 0 & 0 & 0 & | & 0 & 0 & 0 \\ 0 & 0 & 0 & | & 0 & 0 & 0 \end{bmatrix} x = \begin{bmatrix} -1 & 0 & 0 & | & +R_1 & R_1 & 0 \\ 0 & -1 & 0 & | & -R_2 & -R_2 & 0 \\ 0 & 0 & -1 & | & 0 & 0 & R_3 \\ 0 & 0 & 0 & | & -1 & 0 & 0 \\ -1 & 1 & 0 & | & 0 & -R_5 & 0 \\ 0 & -g_2 & 0 & | & 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} R_1$$

$$E \dot{x} = Ax + Bu,$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \begin{matrix} x_1 = \text{input} \\ x_2 = \text{output} \end{matrix}$$

$$y = Cx$$

semi-state eqs. \Rightarrow differential-algebraic eqs.
 singular differential eqs.