

610 Fall 2017 – Homework 4 Due Th 10/05/17

1. (30 points, square integrable functions)

Determine which of the following are square-integrable over the range  $-\infty < t < +\infty$ , that is  $L^2$  functions.

- a)  $1(t)$  = unit step function which is zero for  $t < 0$  and 1 for  $t > 0$
- b) The derivative of  $1(t)$  = unit impulse
- c) The product  $1(t-3) \times 1(4-t)$
- d) The function  $1/t$
- e) The Gaussian  $\exp(-t^2)$

2. (20 points, Fourier of square-integrable)

Show that if  $f(t)$  is square-integrable so is the Fourier transform and vice versa.

How about the convolution,  $f * g$ ?

3. (40 points, Positive-Real and Bounded-Real functions)

For each of the following driving point functions, determine the ranges of the real parameters  $a$  and  $b$  for which they are positive-real and also for which they are bounded-real.

- a)  $f_a(s) = [as-a]/[bs+b]$
- b)  $f_b(s) = [s^2+as+a]/[s^2+bs+b]$
- c)  $f_c(s) = s[s^2+as+1]/[s^2+bs+1][s^2+1]$
- d)  $f_d(s) = s/(s^a + 1)^b$