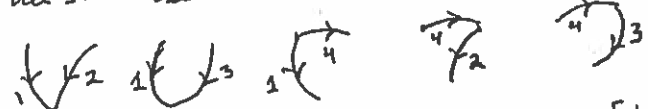
EE610 F2017 Solutions

#1. tree 12 tree 13 tree 14 tree 24 tree 34

a) 

b) $C_{12} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$, $C_{13} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$, $C_{14} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$, $C_{24} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$, $C_{34} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$

Desire $C_{12} = TC_{13} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \Rightarrow 1 = t_{11},\ 0 = t_{12} = 0,\ -1 = -t_{11} + t_{12}$

$0 = t_{21},\ 1 = t_{22} = 1,\ 1 = -t_{21} + t_{22}$

$\Rightarrow t_{11} = 1,\ t_{12} = 0 \Rightarrow$ agrees $-1 = -1 + 0 \Rightarrow T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1_2$ as $C_{12} = C_{13} \Rightarrow T^{-1} = 1_2$

$t_{21} = 0,\ t_{22} = 1 \Rightarrow 1 = 0 + 1$

#2. As the poles are all on the $j\omega$ axis, they must be simple with positive residues

$\therefore\ y(s) = \dfrac{k_1 s}{s^2+1} + \dfrac{k_2 s}{s^2+4} = \dfrac{k_1 s^3 + 4k_1 s + k_2 s^3 + k_2 s}{(s^2+1)(s^2+4)} = \dfrac{s[(k_1+k_2)s^2 + (4k_1+k_2)]}{(s^2+1)(s^2+4)}$

$\Rightarrow k_1 + k_2 = 1,\ 4k_1 + k_2 = a \Rightarrow k_2 = 1 - k_1 \Rightarrow 4k_1 + (1-k_1) = a \Rightarrow a = 3k_1 + 1$

$\therefore\ a = 3k_1 + 1$ for $k_1 > 0 \Rightarrow 1 < a$ and $k_2 > 0 \Rightarrow k_1 < 1 \Rightarrow a < 3+1 = 4$; $\therefore\ 1 \le a \le 4$

which is the same as poles & zeros alternate (cancellation if $a = 1$ or 4)

For Cauer synthesis $y(s) = \dfrac{s^3 + as}{s^4 + 5s^2 + 4} \Rightarrow$

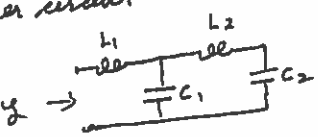
$s^3 + as \,)\, s^4 + 5s^2 + 4 \,(\, s$; $s^4 + as^2$; $(5-a)s^2 + 4 \,)\, s^3 + as \,(\, \frac{1}{5-a}s$; $s^3 + \frac{4}{5-a}s$; $\frac{5a-a^2-4}{5-a}s \,)\, (5-a)s^2 + 4 \,(\, \frac{(5-a)^2}{5a-a^2-4}s$; $(5-a)s^2$; $4 \,)\, \frac{5a-a^2-4}{5-a}$

$\therefore\ y(s) = \cfrac{1}{s + \cfrac{1}{\frac{1}{5-a}s + \cfrac{1}{\frac{(5-a)^2 s}{5a-a^2-4} + \cfrac{1}{\frac{5a-a^2-4}{4(5-a)}s}}}}$

all coefficients are > 0 for $1 < a < 4$

$(5a - a^2 - 4) = (4-a)(1+a)$

$\therefore$ Cauer circuit



$L_1 = 1,\ L_2 = \dfrac{(5-a)^2}{5a-a^2-4}$

$C_1 = \dfrac{1}{5-a},\ C_2 = \dfrac{5a-a^2-4}{4(5-a)}$

(some absent if $a = 1$ or 4)

#3. $S_1 = \dfrac{2s^2}{2s^2+as+b}$, $S_2 = \dfrac{1}{2s^2+as+b}$, $D = 2s^2 + as + b$ must be Hurwitz & $|S_i(j\omega)| \le 1$, i

$\Rightarrow a > 0$ & $b > 0$

For $S_2(0) = 1/b \le 1 \Rightarrow b \ge 1$

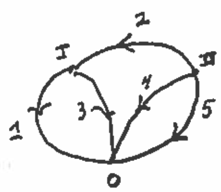
For $|S_1(j\omega)|^2 \le 1 \Rightarrow 4\omega^2 \le (b - 2\omega^2)^2 + (a\omega)^2 \Rightarrow 0 \le b^2 + (a^2 - 4b)\omega^2 \Rightarrow$ as $\omega \to \infty \Rightarrow a^2 - 4b$

For $|S_2(j\omega)|^2 \le 1 \Rightarrow 1 \le b^2 + (a^2-4b)\omega^2 + 4\omega^4$ is true for $a^2 - 4b \ge 0$, $b \ge 1$

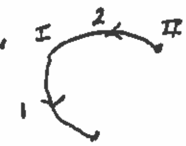
when $a^2 \ge 4b \ge 4$

$\therefore\ S_1$ & S_2 are BR as $S_i^*(s) = S_i(s^*)$, $i = 1, 2$,

#4.

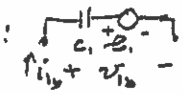


use tree as



$$e=\begin{bmatrix}1 & 0 & 1 & 1 & 1\\ 0 & 1 & 0 & 1 & 1\end{bmatrix},\quad \mathcal{T}=\begin{bmatrix}-1 & 0 & 1 & 0 & 0\\ -1 & -1 & 0 & 1 & 0\\ -1 & -1 & 0 & 0 & 1\end{bmatrix};\quad e^T v_t = v_b,\ \mathcal{T}^T i_\ell = i_b$$
$= v+e$ $= i+j = i'$ here

$AV=BE$ for branch 1:



$i_{1b} = i_1 = sCv_1,\quad v_{1b} = v_1 + e_1 \Rightarrow v_1 = v_{1b} - e_1$

$$\begin{bmatrix}sC_1 & & & & \\ & sC_2 & & & \\ & & 0 & 0 & \\ & & g_m & 0 & \\ & & & & G_5\end{bmatrix} v = \begin{bmatrix}1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1\end{bmatrix} i' = 1_5\cdot \mathcal{T}^T i_\ell$$

$= Y_{brs}\cdot[e^T v_t - e^T e]$

$$=\begin{bmatrix}sC_1 & & & & \\ & sC_2 & & & \\ & & 0 & 0 & \\ & & g_m & 0 & \\ & & & & G_5\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\\ 1 & 0\\ 1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}v_{C_1}\\ v_{C_2}\end{bmatrix} - \begin{bmatrix}sC_1e_1\\ 0\\ 0\\ 0\\ 0\end{bmatrix} = \begin{bmatrix}-1 & -1 & -1\\ 0 & -1 & -1\\ 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}i_3\\ i_4\\ i_5\end{bmatrix}$$

$$=\begin{bmatrix}C_1 & 0 & 0 & 0 & 0\\ 0 & C_2 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\dot v_t\\ \dot i_\ell\end{bmatrix} = \begin{bmatrix}0 & 0 & -1 & -1 & -1\\ 0 & 0 & 0 & -1 & -1\\ 0 & 0 & 0 & 0 & 0\\ -g_m & 0 & 0 & 0 & 0\\ 0 & -G_5 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}v_t\\ i_\ell\end{bmatrix} + \begin{bmatrix}C_1\dot e_1\\ 0\\ 0\\ 0\\ 0\end{bmatrix}\qquad \begin{bmatrix}v_t\\ i_\ell\end{bmatrix} = \begin{bmatrix}v_{1b}\\ v_2\\ i_3\\ i_4\\ i_5\end{bmatrix} = x$$

$$y = v_5 = [0\,0\,0\,0\,1]e^T v_t = [v_{C_1}+v_{C_2}] = [1\ 1\ 0\ 0\ 0]x$$

Can use this to get v_5/e_1, but it is not semi-state due to $\dot{}$ on e_1 (can pull branch 1 as two branches)

so define a new variable $x_6 = e_1 = u \Rightarrow \hat x = \begin{bmatrix}x\\ x_6\end{bmatrix} \Rightarrow$

$$\begin{bmatrix}C_1 & 0 & 0 & 0 & 0 & -C_1\\ 0 & C_2 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\dot{\hat x} = \begin{bmatrix}0 & 0 & -1 & -1 & -1 & -1\\ 0 & 0 & 0 & -1 & -1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ -g_m & 0 & 0 & 0 & 0 & 0\\ 0 & -G_5 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & -1\end{bmatrix}\hat x + \begin{bmatrix}0\\ 0\\ 0\\ 0\\ 0\\ 1\end{bmatrix}u,\quad y = [1\ 1\ 0\ 0\ 0\ 0]\hat x$$

Can reduce this to get $v_5/e_1 = (\hat x_1 + \hat x_2)/u$.

Easier: $-i_{C_1} = -i_{C_2} = sC_1(e_1 - v_I) = sC_2(v_I - v_{II})$ & $i_{R_5} = -i_{C_2} - g_m v_I = G_5 v_{II}$

$\Rightarrow sC_1e_1 - (sC_1 + sC_2)v_I = -sC_2 v_{II}$

$\Rightarrow v_I = (sC_2 v_{II} + sC_1 e_1)/(s[C_1+C_2]) \Rightarrow G_5 v_{II} = sC_1e_1 - (sC_1 + g_m)\dfrac{[sC_1e_1 + sC_2 v_{II}]}{s(C_1+C_2)}$

$= \dfrac{[s^2C_1^2 + s^2C_1C_2 - s^2C_1^2 - sC_1 g_m]e_1 - (sC_1+g_m)\cdot sC_2 v_{II}}{s(C_1+C_2)}$

as $y = v_5 = v_{II}$

$\Rightarrow \big((G_5)[s(C_1+C_2)] + [sC_1+g_m]\cdot sC_2\big)y = sC_1[sC_2 - g_m]e_1$

$$\Rightarrow \frac{y}{e_1} = \frac{sC_1[sC_2-g_m]}{s[(C_1+C_2)G_5 + sC_1C_2 + g_m C_2]} = \frac{C_1[sC_2 - g_m]}{sC_1C_2 + [g_m C_2 + G_5(C_1+C_2)]} = \frac{s - g_m/C_2}{s + \left[\dfrac{g_m}{C_1} + G_5\left(1+\dfrac{C_2}{C_1}\right)\right]}$$