

$$= \sqrt{2\beta I_T} v_d \sqrt{1 - \left(\frac{\beta v_d^2}{2 I_T}\right)} \Rightarrow 2\beta I_T v_d^2 \left(1 - \frac{\beta v_d^2}{2 I_T}\right) = I_T^2$$

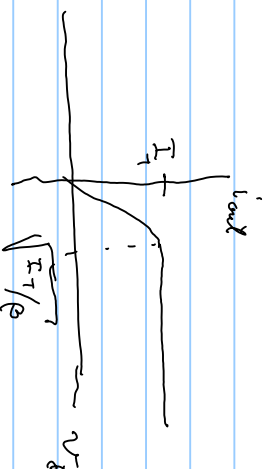
$$-\beta^2 v_d^4 + 2\beta I_T v_d^2 - I_T^2 = 0$$

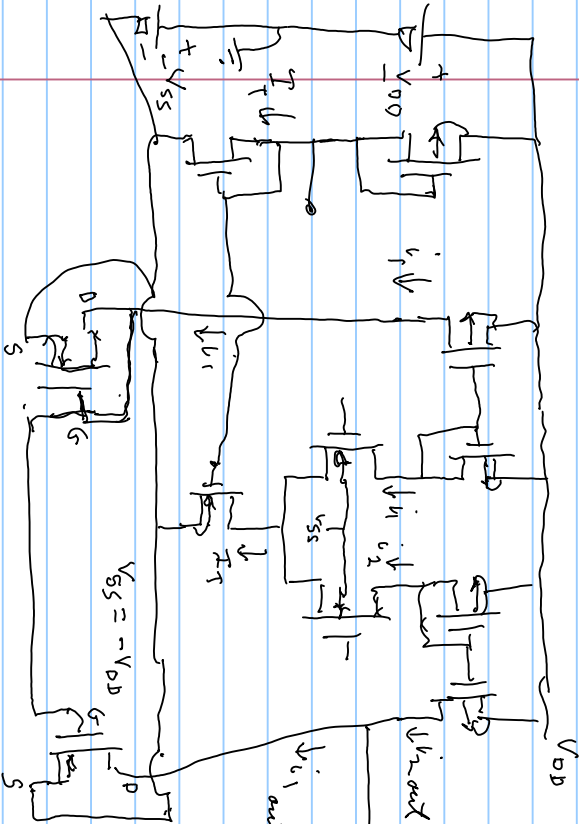
$$x^2 - \left(\frac{2 I_T}{\beta}\right)x + \left(\frac{I_T}{\beta}\right)^2 = 0$$

$$x = v_d^2$$

$$v_d = \pm \sqrt{\frac{I_T}{\beta}}$$

$$x = \frac{I_T}{\beta} \pm \frac{1}{2} \sqrt{\underbrace{\left(\frac{2 I_T}{\beta}\right)^2 - 4 \left(\frac{I_T}{\beta}\right)^2}_0} = \frac{I_T}{\beta} \pm v_d^2$$





$i_{out} = i_{in1} - i_{in2}$
 if open drain voltage of output
 transistors adjust to give $i_{out} = i_{in}$
 and v_{out}

for MOS

$$i_{out} = (a_{gm} v_d) \cdot I_T \left\{ \begin{array}{l} 1 \\ \frac{\sqrt{2} v_d}{I_T} \sqrt{1 - \left(\frac{\sqrt{2} v_d}{I_T} \right)^2} \end{array} \right. \quad \begin{array}{l} v_d \geq (I_T / \theta)^{1/2} = V_D \\ -V_D < v_d < V_D \end{array}$$

$$v_d \leq -(I_T / \theta)^{1/2} = -V_D$$

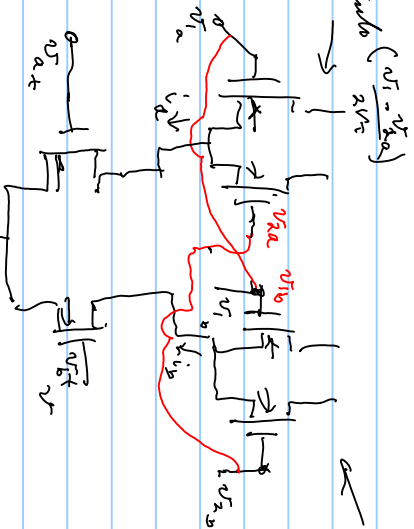
for BJT

$$i_{out} = I_T \tanh\left(\frac{v_d}{2 V_T}\right)$$

to make a multiplier use two I_T circuits feeding two OTHs

$$i_{out} = I_T \tanh\left(\frac{v_{a1}/2V_T}{1}\right) \cdot \tanh\left(\frac{v_{a2}/2V_T}{1}\right) \approx I_T \cdot \frac{v_{a1}}{2V_T} \cdot \frac{v_{a2}}{2V_T} \quad \text{gives a 4-quadrant multiplier}$$

$$i_{a1} \approx i_{a2} \tanh\left(\frac{v_T - v_{a0}}{2V_T}\right)$$



$$i_{o1} = i_{b1} \tanh\left(\frac{v_T - v_{a1}}{2V_T}\right)$$

$$i_{o1} - i_{o2} = i_{b1} \tanh\left(\frac{v_T - v_{a1}}{2V_T}\right) - i_{b2} \tanh\left(\frac{v_T - v_{a2}}{2V_T}\right)$$

$$i_{a1} - i_{a2} = I_T \tanh\left(\frac{v_T - v_{a1}}{2V_T}\right)$$

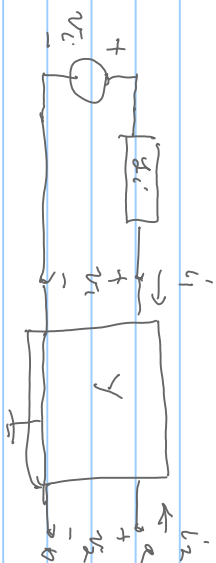
$$i_{o1} - i_{o2} = (i_{a1} - i_{a2}) \tanh\left(\frac{v_T - v_{a1}}{2V_T}\right)$$

$$\approx I_T \tanh\left(\frac{v_{a1} - v_{a2}}{2V_T}\right) \cdot \tanh\left(\frac{v_{a1} - v_{a2}}{2V_T}\right)$$

$\Rightarrow$ a 4-quadrant multiplier

But multiplier $\Rightarrow$ small signal $i_o = I_T \cdot \frac{v_{a1}}{2V_T} \cdot \frac{v_{a2}}{2V_T}$

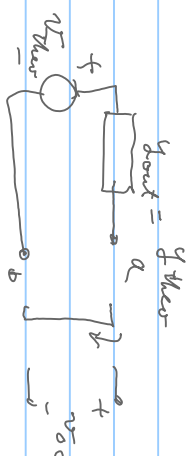
$$v_{a1} = v_{a1} - v_{a2}$$



$$i_1 = y_{11} v_1 + y_{12} v_2$$

$$i_2 = y_{21} v_1 + y_{22} v_2$$

i_2 short circuit current
 v_2 open circuit voltage



$$i_2 \text{ short} \Rightarrow v_2 = 0$$

$$i_2 \text{ short} = y_{21} v_1, \quad i_1 = y_{11} v_1$$

$$v_2 \text{ open}, \quad i_2 = 0 \Rightarrow$$

$$y_{21} v_1 = -y_{22} v_2 \text{ open} : y_{21} (v_1 - v_2) = i_1$$

$$i_1 = y_{11} v_1 + y_{12} v_2 = y_{11} v_1 - y_{12} v_1 = y_{11} v_1 + y_{12} v_2$$

$$v_2 \text{ open} = -\frac{y_{21} v_1}{y_{22}}$$

$$(y_{11} + y_{12}) \frac{y_{22} v_1}{-y_{21}} = y_{11} v_1 - y_{12} v_2$$

$$\Rightarrow (y_{11} y_{22} - y_{12} y_{21} + y_{22} y_{12}) v_1 = y_{11} v_1$$

$$= \frac{y_{11} + y_{12} y_{22}}{-y_{21}} v_1$$

$$v_2 \text{ open} = -\frac{y_{12} y_{21}}{y_{11} + y_{12} y_{21}} v_1$$

