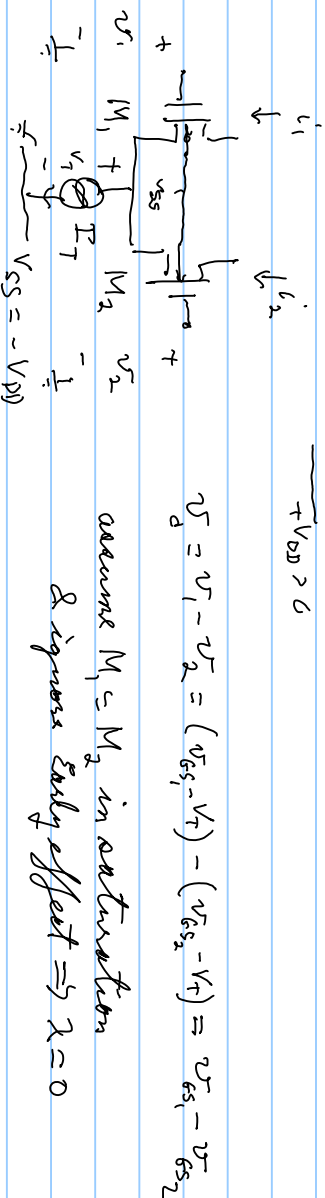


TI watchChronos: ez430-Chronos Development Tool

Homework 5 due next Tu

OTA's with CMOS, NMOS diff. pairs



$$v_d = v_1 - v_2 = (v_{GS1} - V_T) - (v_{GS2} - V_T) = v_{GS1} - v_{GS2}$$

$$i_1 = \frac{k_p}{2} \frac{W}{L} (v_{GS1} - V_{th})^2, \quad i_2 = \frac{k_p}{2} \frac{W}{L} (v_{GS2} - V_{th})^2$$

$$= \beta (v_{GS1} - V_{th})^2, \quad i_2 = \beta (v_{GS2} - V_{th})^2, \quad i_1 + i_2 = I_T$$

$$v_{GS1} = V_{th} + \sqrt{i_1/\beta}, \quad v_{GS2} = V_{th} + \sqrt{i_2/\beta} \Rightarrow \text{need + signs as assume } M_1 \& M_2 \text{ are on}$$

$$v_d = v_{d1} - v_{d2} = \sqrt{\frac{v_1^2}{\rho}} - \sqrt{\frac{v_2^2}{\rho}} \Rightarrow v^2 = \frac{v_1^2}{\rho} + \frac{v_2^2}{\rho} - 2 \sqrt{\frac{v_1 v_2}{\rho}} = \frac{I_T - 2}{\rho} \sqrt{v_1 v_2}$$

$$\left(\frac{I_T}{\rho} - v_d^2 \right)^2 = \left(\frac{2}{\rho} \sqrt{v_1 v_2} \right)^2 \Rightarrow v_1 v_2 = \frac{\rho^2}{4 \rho^2} \left(I_T - \rho v_d^2 \right)^2 \quad \& \quad v_2 = I_T - v_1$$

$$v_1 \cdot (I_T - v_1) = \frac{1}{4} (I_T - \rho v_d^2)^2 \Rightarrow v_1^2 - I_T v_1 + \frac{1}{4} (I_T - \rho v_d^2)^2 = 0$$

$$v_1 = \frac{I_T \pm \frac{1}{2}}{2} \sqrt{\frac{I_T^2 - \frac{4}{4} (I_T - \rho v_d^2)^2}{2}} = \frac{I_T \pm \frac{1}{2}}{2} \sqrt{I_T^2 - (I_T^2 - 2 \rho I_T v_d^2 + \rho^2 v_d^4)} \\ = \frac{I_T \pm \frac{1}{2}}{2} \sqrt{2 \rho I_T v_d^2 - \rho^2 v_d^4} = \frac{I_T \pm \frac{I_T}{2}}{2} \sqrt{\left(2 \rho \frac{v_d^2}{I_T} \right) - \frac{\rho^2 v_d^4}{I_T^2}}$$

$$= \frac{I_T}{2} \left(1 \pm \sqrt{\frac{2 \rho v_d^2}{I_T}} \sqrt{1 - \left(\frac{\rho}{2} \frac{v_d^2}{I_T} \right)} \right) = \frac{I_T \pm \frac{I_T}{2}}{2} \sqrt{\frac{2 \rho}{I_T}} \cdot v_d \sqrt{1 - \left(\frac{\rho}{2} \frac{v_d^2}{I_T} \right)}$$

$$\text{near } v_d = 0 \Rightarrow v_1 = \frac{I_T}{2} \pm \sqrt{\frac{\rho}{2} I_T} \cdot v_d \approx \text{small signal } v_1 = \pm \sqrt{\frac{\rho}{2} I_T} \cdot v_d$$

$$\Rightarrow v_2 = \frac{I_T}{2} - \sqrt{\frac{I_T \rho}{2}} \cdot v_d \sqrt{1 - \left(\frac{\rho}{2} \frac{v_d^2}{I_T} \right)} \approx \frac{I_T}{2} - \sqrt{\frac{\rho}{2} I_T} \cdot v_d$$

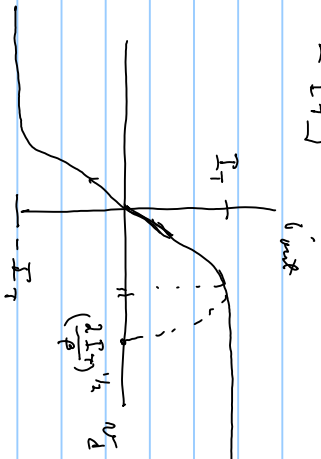
$$i_{out} = i_1 - i_2 \Rightarrow \text{small signal } i_{out} = \left\{ \frac{\beta}{2} I_T - (1) \left(\frac{\beta}{2} I_T \right) \right\} v_d = \left[\frac{\beta}{2} I_T \right] v_d = \beta_m v_d$$

$$= \frac{I_T + \left[\frac{\beta}{2} I_T v_d \right] \left[1 - \left(\frac{\beta}{2} \frac{v_d^2}{I_T} \right) \right] - \left[\frac{I_T}{2} - \left[\frac{\beta}{2} I_T v_d \right] \left[1 - \frac{\beta}{2} \frac{v_d^2}{I_T} \right] \right]}{2} = \sqrt{2\beta I_T} v_d \sqrt{1 - \left(\frac{\beta}{2} \frac{v_d^2}{I_T} \right)}$$

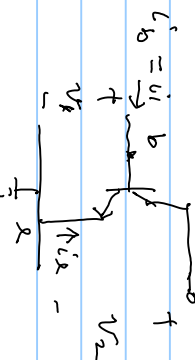
$$\beta_m = \sqrt{2\beta I_T}$$

find v_d such that $i_{out} = I_T$
 can find this v_d by solving

$$I_T^2 = 2\beta I_T v_d^2 \left(1 - \left(\frac{\beta}{2} \frac{v_d^2}{I_T} \right) \right)$$

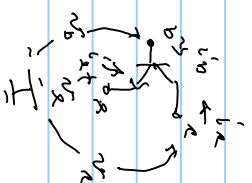


$$KCL \quad i'_x + i'_b + i'_c = 0 \quad i'_x = -i'_b - i'_c$$



$$\begin{bmatrix} i'_b \\ i'_c \\ i'_x \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} v_b \\ v_c \end{bmatrix} \quad \begin{bmatrix} v_b \\ v_c \end{bmatrix} = \begin{bmatrix} v_d \\ v_d \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} i'_b \\ i'_c \\ i'_x \end{bmatrix} = \begin{bmatrix} g_n + a_{gn} & 0 & -g_n - a_{gn} \\ g_m & g_o & -g_m - g_o \\ -g_n - a_{gn} & -g_o & g_n + a_{gn} \end{bmatrix} \begin{bmatrix} v_b \\ v_c \\ v_x \end{bmatrix}$$



ground collector $\Rightarrow v_c = 0$ & ignore $i_c = -i_x - i_b \Rightarrow$ write out i_x & $v_c \Rightarrow 2$ unknowns

$$\begin{bmatrix} i_b \\ i_x \end{bmatrix} = \begin{bmatrix} g_1 + \beta g_m & -g_m - \beta g_m \\ -g_m - \beta g_m & g_m + \beta g_m \end{bmatrix} \begin{bmatrix} v_b \\ v_x \end{bmatrix}$$

