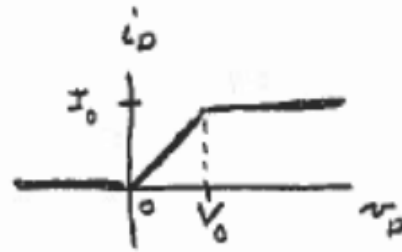
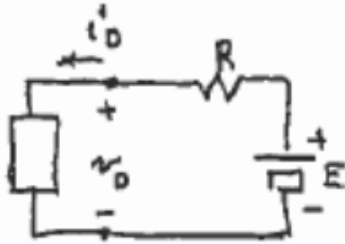


HW 2

1. (50 points, Q points and load lines) The following circuit has the 1-port diode described by the piecewise linear curve shown.



a) Assume $E > V_0 > 0$ and $R > 0$. Give a range of R to the Q point is in the positive slope region; repeat for the Q point in the constant current $= I_0$ region. Draw separate graphs for typical load lines on the diode curve for each of the two cases.

Kirchhoff's Voltage Law gives:

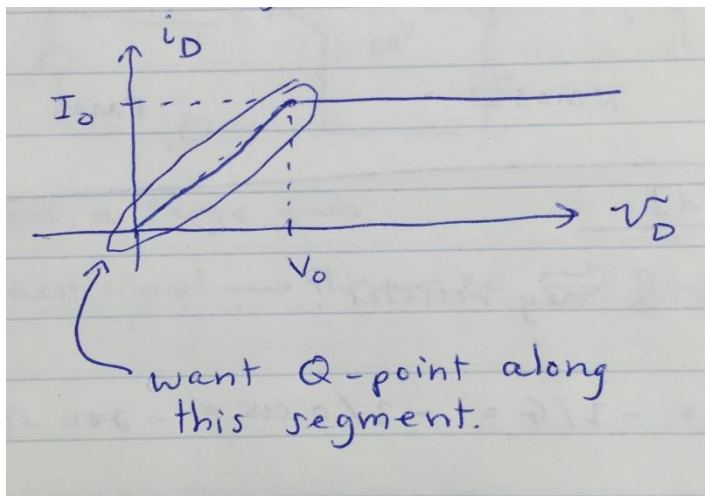
$$E = V_R + V_D$$

$$E = i_D R + V_D$$

Rearranging this gives...

$$i_D = (E - V_D)/R$$

Notice that this load line equation will be a diagonal line with a negative slope that intersects the diode i-v characteristic at different points, depending on the value of R .



LOWER BOUND

R (least resistance) will have the load line intersect the diode curve at (V_o, I_o) .

$$I_o = (E - V_o)/R \quad \text{and solving for } R \dots \quad R = (E - V_o)/I_o$$

UPPER BOUND

R (most resistance) will have the load line intersect the diode curve at $(0, 0)$. Which means that no current will be passing through the circuit.

$$R = (E-0)/0 \dots \quad R = E/0, \text{ which is } \infty.$$

This result makes sense: an infinitely resistive circuit element would be needed to stop the voltage source (E) from sending any current through the circuit.

So the final answer: R must have a minimum value of $(E - V_o)/I_o$ [Ω] to keep the Q-point in the positive slope region.

R must have a MAX value of $(E - V_o)/I_o$ [Ω] for the Q-point to be in the constant current region (the minimum of course would be zero).

b) Give the small signal conductance in the above two regions. Explain what happens if Q is at the two break points, $v_D=0$ & V_0 .

Note that the small signal approximation for conductance is

$g_D = \frac{dI}{dV}$ at point Q. So the conductance will merely be the slope of the diode curve.

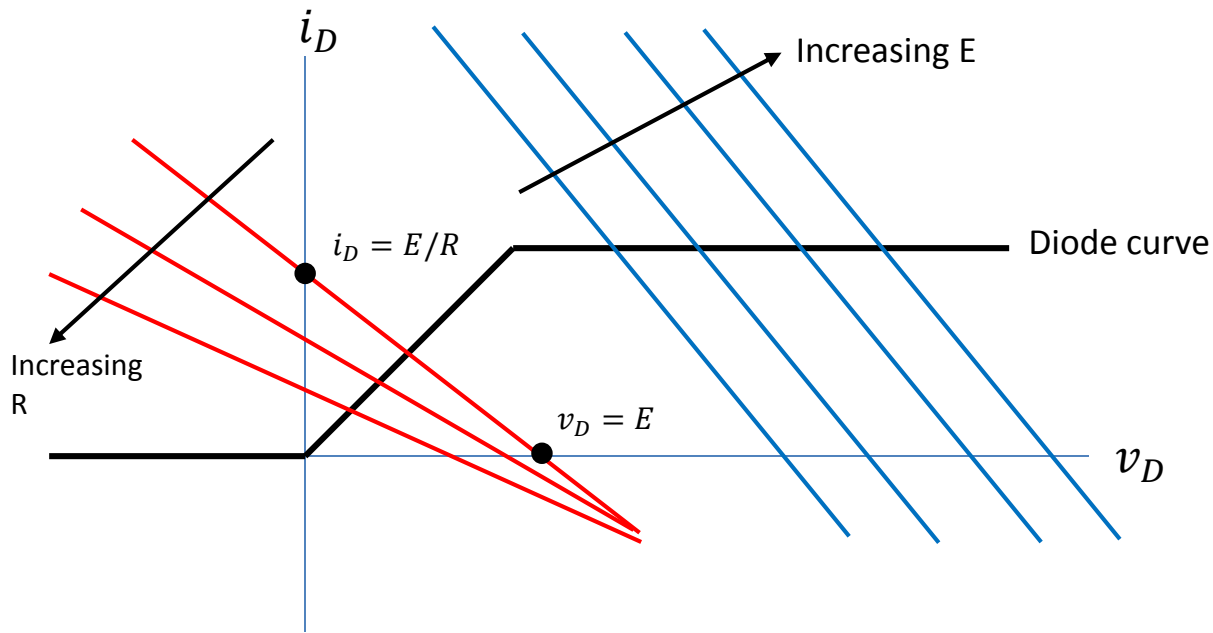
In the positive-slope region: $g_D = \frac{I_0}{V_0}$

In the constant-current region: $g_D = 0$

At the junction between regions, the curve would not have a defined derivative with respect to voltage. In reality, such a discontinuity would not exist in a diode's behavior, and we could assume it is continuous over a very small interval. Somewhere along the continuous junction there is a point that is the average of the conductance on either side: we could approximate it as the average of the two slopes on either side of the junction.

$g_D = \frac{\frac{I_0}{V_0} + 0}{2} = \frac{I_0}{2V_0}$ at the (V_0, I_0) junction, for example

You could also argue that g_D is undefined at the junctions, because the discontinuities are not differentiable.



The red lines are example load lines in the positive slope region.
The blue lines are example load lines in the constant current region.

c) Next assume that $E < 0$ and $R < 0$. Pictorially with three different graphs show that there are one, two or three (that is, multivalued) Q points and discuss what you think this means.

Recall that the load line is $i_D = \frac{E - V_D}{R}$.

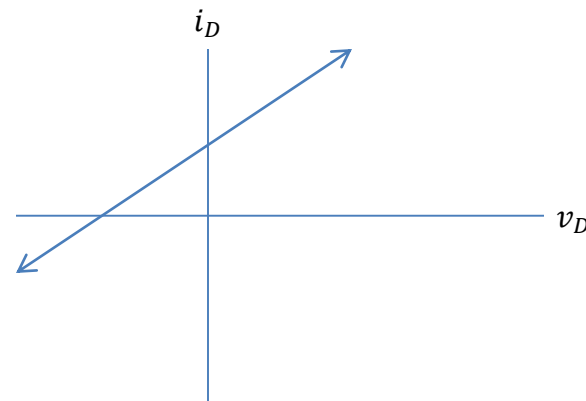
Let's break up the two terms:

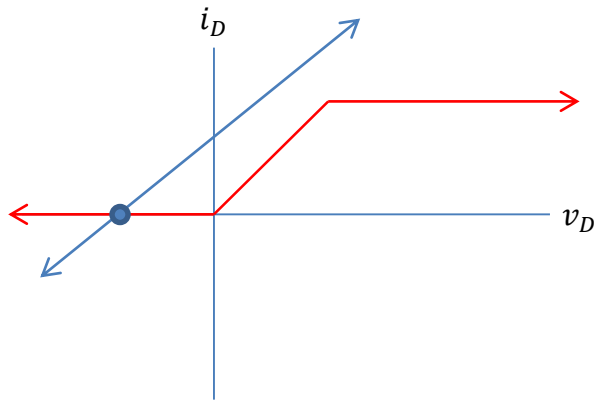
$$i_D = \frac{E}{R} + \left(-\frac{V_D}{R} \right)$$

This (y-intercept) will always be positive because E and R have the same sign

This term has a positive slope equal to $\frac{-1}{-R}$

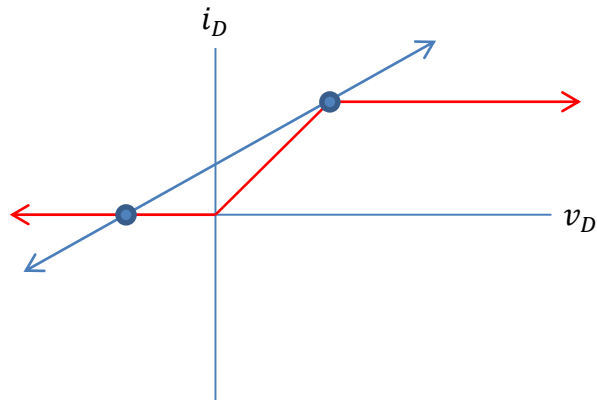
So the load line will generally look like this:





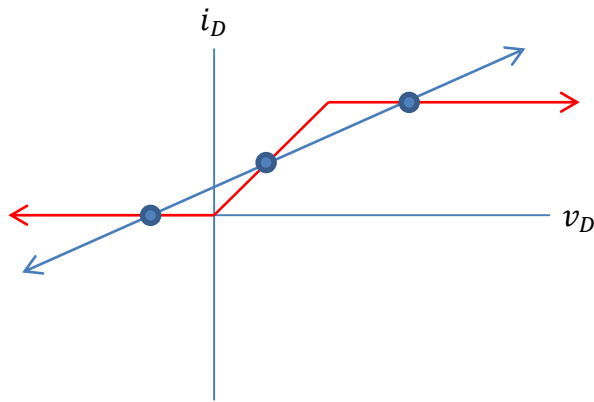
ONE Q-POINT

The conceptualization of this is pretty straight-forward. The diode operates with a negative voltage (relative to ground) but does not allow any current to pass through.



TWO Q-POINTS

This can happen if your load line intersects one of the junctions in the diode curve (either at $(0,0)$ or (V_o, I_o)). If your load line intersects the origin, it means that the voltage source (E) is zero, so the only power entering the circuit is from the negative resistor. Two Q-points cannot exist simultaneously (the diode cannot have two voltage values or two current values), but there are two possible equilibriums for the circuit. The initial conditions may affect which equilibrium is reached.



THREE Q-POINTS

This happens if your load line intersects all three piecewise regions of the diode characteristic: one Q-point in the negative-voltage region, one Q-point in the positive-slope region, and one Q-point in the constant-current (positive-voltage) region.

d) As the diode characteristic is piece-wise linear it can be written in one formula as a sum of unit-step functions with coefficients. Obtain such a representation.

Recall that the Heaviside function, $H(x)$, (or unit step function) has a value of 0 until $x=0$, and then immediately jumps to 1, where it stays for all greater values of x .

Similarly, $kH(x - a) + b$ begins at value b and then jumps up to $(b+k)$ at $x=a$.

Also note that we can create a ramp function with a slope of k by multiplying the modified Heaviside function above by its independent variable (which is x in this case):

$$kR(x) := kxH(x - a) + b$$

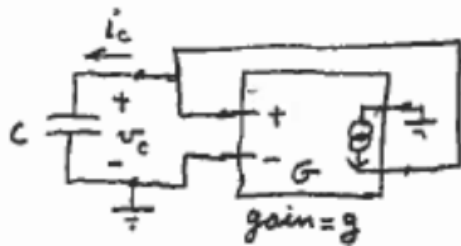
So we want to create the composite of two functions:

- A Heaviside ramp with slope $\frac{I_0}{V_0}$ that begins at $v_D = 0$
- A Heaviside ramp with equal and opposite slope $-\frac{I_0}{V_0}$ that begins at $v_D = V_0$. This will cancel out the slope of the first function, so we can get a constant current line after V_0 .

And we get:

$$i_D = \left(\frac{I_0}{V_0}\right)v_D H(v_D) - \left(\frac{I_0}{V_0}\right)(v_D - V_0)H(v_D - V_0)$$
$$i_D = \left(\frac{I_0}{V_0}\right)v_D H(v_D) - \left(\frac{I_0}{V_0}\right)(v_D)H(v_D - V_0) + I_0 H(v_D - V_0)$$

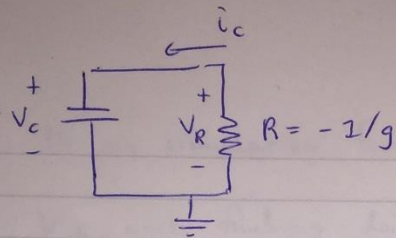
2. (50 points, Spice G and load line)



- a) Set up the differential equation for the above circuit which uses the G (=VCCS) component of Spice. Solve the differential equation for $t > 0$ when the capacitor initial condition is $V_c(0) > 0$ and again when $V_c(0) = 0$ and $g > 0$. But note even though the G component gain may be positive its input resistance can be negative. G components can though have negative gains.

Does the set-up of the G component look familiar? It should, it is simply acting as a negative resistor. The current passing through the device is equal to the voltage across it multiplied by $-g$:

$$R = \frac{V}{I} = \frac{V}{-gV} = \frac{-1}{g}$$



$$\text{KVL}$$

$$V_c = V_R$$

Note: because of sign convention,
 $i_c = -i_R$

$$V_c = i_R R = (-i_c)(-1/g)$$

$$V_c = i_c / g$$

$$i_c = C \frac{dV_c}{dt}$$

$$V_c = \left(\frac{C}{g} \right) \frac{dV_c}{dt}$$

$$\int_{V_{c,0}}^{V_c} \frac{dV_c}{V_c} = \int_0^t \left(\frac{g}{C} \right) dt$$

τ is the time constant for the RC circuit

$$\tau = C/g = |R|C$$

$$\ln(V_c) - \ln(V_{c,0}) = t/\tau$$

$$\ln(V_c/V_{c,0}) = t/\tau$$

$$[V_c/V_{c,0}] = e^{t/\tau}$$

$$V_c = (V_{c,0}) e^{t/\tau}$$

The voltage across the capacitor increases exponentially!

There are 3 cases:

(A) $V_{c,0} > 0$ ----- V_c goes to ∞

(B) $V_{c,0} = 0$ ----- The circuit does nothing if there is no initial charge on the capacitor.

(C) $V_{c,0} < 0$ ----- V_c goes to $-\infty$