

file: u:/matrix_exp.mcd RWN 09/10/14

means to take exponential (or other functions, $f(\cdot)$) of a matrix by obtaining eigenvalues and eigenvectors for distinct eigenvalues. $M = v \cdot e \cdot v^{-1}$; $f(M) = v \cdot f(e) \cdot v^{-1}$.

$$M := \begin{pmatrix} 2 & -1 & 3 \\ -1 & 2 & -2 \\ -3 & -2 & 2 \end{pmatrix}$$

$$e(M) := \text{eigenvals}(M)$$

$$e(M) = \begin{pmatrix} -1.202 \\ 1.089 \\ 6.113 \end{pmatrix}$$

$$\text{ediag}(M) := \text{diag}(e(M))$$

$$\text{veig}(M) := \text{eigenvecs}(M)$$

$$\text{ediag}(M) = \begin{pmatrix} -1.202 & 0 & 0 \\ 0 & 1.089 & 0 \\ 0 & 0 & 6.113 \end{pmatrix}$$

$$\text{veig}(M) = \begin{pmatrix} -0.611 & 0.525 & 0.593 \\ 0.273 & 0.843 & -0.464 \\ 0.743 & 0.121 & 0.658 \end{pmatrix}$$

$$\text{veig}(M)^{-1} = \begin{pmatrix} -0.611 & 0.273 & 0.743 \\ 0.525 & 0.843 & 0.121 \\ 0.593 & -0.464 & 0.658 \end{pmatrix}$$

$$M_{\text{new}}(M) := \text{veig}(M) \cdot \text{ediag}(M) \cdot \text{veig}(M)^{-1}$$

To check that $M_{\text{new}}(M) = M$; note if $\text{veig}(M)$ and its inverse are reversed this gives the wrong result

$$M_{\text{new}}(M) = \begin{pmatrix} 2 & -1 & 3 \\ -1 & 2 & -2 \\ 3 & -2 & 2 \end{pmatrix}$$

$$\text{expl}(M) := \exp(e(M))$$

$$\text{expl}(M) = \begin{pmatrix} 0.301 \\ 2.971 \\ 451.733 \end{pmatrix}$$

$$\text{diagexp}(M) := \text{diag}(\text{expl}(M))$$

$$\text{diagexp}(M) = \begin{pmatrix} 0.301 & 0 & 0 \\ 0 & 2.971 & 0 \\ 0 & 0 & 451.733 \end{pmatrix}$$

$$\text{expM}(M) := \text{veig}(M) \cdot \text{diagexp}(M) \cdot \text{veig}(M)^{-1}$$

$$\text{expM}(M) = \begin{pmatrix} 159.702 & -123.039 & 176.3 \\ -123.039 & 99.447 & -137.618 \\ 176.3 & -137.618 & 195.855 \end{pmatrix}$$

The above does not work when there are multiple eigenvalues as seen by the following.

$$M2 := \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}$$

$$\text{expM}(M2) = \begin{pmatrix} 7.389 & 0 \\ 0 & 7.389 \end{pmatrix}$$

$$\text{eigenvecs}(M2) = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

For equal eigenvalues, the process needs 1's inserted off the diagonal in the