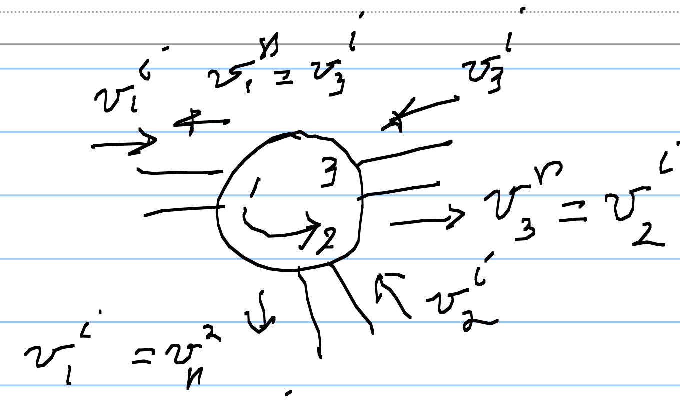
EE610
10/28/14

circulator

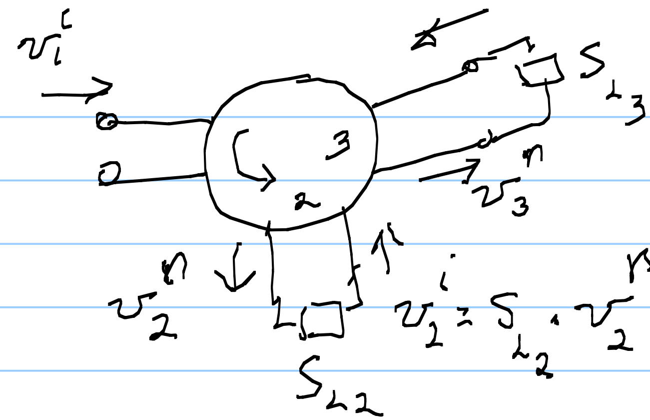


$$\begin{bmatrix} v_1^r \\ v_2^r \\ v_3^r \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v_1^i \\ v_2^i \\ v_3^i \end{bmatrix} \Longrightarrow S = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$S S^T = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 1_3$$

$\Rightarrow$ lossless

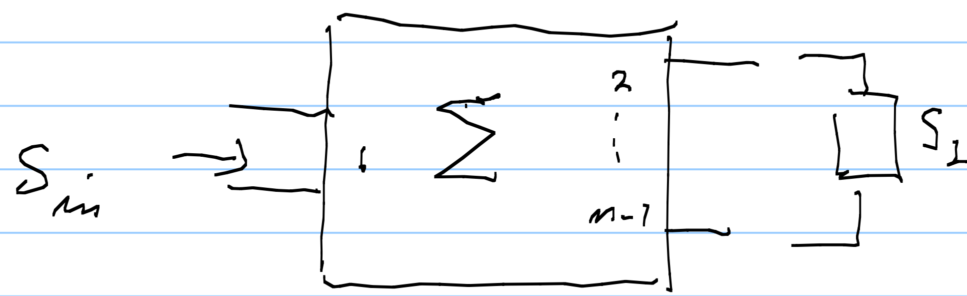
$v_3^i = S_{L_3} \cdot v_3^r = S_{L_3} \cdot v_2^i = S_{L_3} S_{L_2} \cdot v_1^i$



$v_2^i = S_{L_2} \cdot v_2^r = S_{L_2} \cdot v_1^i$

$v_1^r = v_3^i = S_{L_3} \cdot S_{L_2} \cdot v_1^i \Rightarrow S_{in} = S_{L_3} \cdot S_{L_2}$

also that if S_{L_2} & S_{L_3} are BR then S_{in} is BR



$$\begin{bmatrix} v_1^r \\ v_2^r \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} v_1^i \\ v_2^i \end{bmatrix} \Rightarrow \left[\begin{array}{c|cc} \overset{S_{11}}{0} & \multicolumn{2}{c}{\overbrace{0 \quad 1}^{S_{12}}} \\ \hline \underset{S_{21}}{\begin{matrix}1\\0\end{matrix}} & \multicolumn{2}{c}{\underbrace{\begin{matrix}0&0\\1&0\end{matrix}}_{S_{22}}} \end{array}\right]$$

$$v_L^r = S_L v_L^i =$$

$$v_L^i = v_2^r \,, \quad v_L^r = v_2^i$$

$$v_L^i = S_{21} v_1^i + S_{22} v_2^i = \underbrace{S_{21} v_1^i + S_{22} S_L v_2^r = v_2^r}$$

$$(1 - S_{22} S_L) v_2^r = S_{21} v_1^i$$

$$v_1^r = S_{11} v_1^i + S_{12} S_L \overset{v_2^r = v_h}{v_2^r} = S_{11} v_1^i + S_{12} S_L (1 - S_{22} S_L)^{-1} S_{21} v_1^i$$

$$S_{in} = S_{11} + S_{12} S_L \left(I - S_{22} S_L\right)^{-1} S_{21}$$

Ex: Σ = circulator (3-port); $S_{11} = 0$

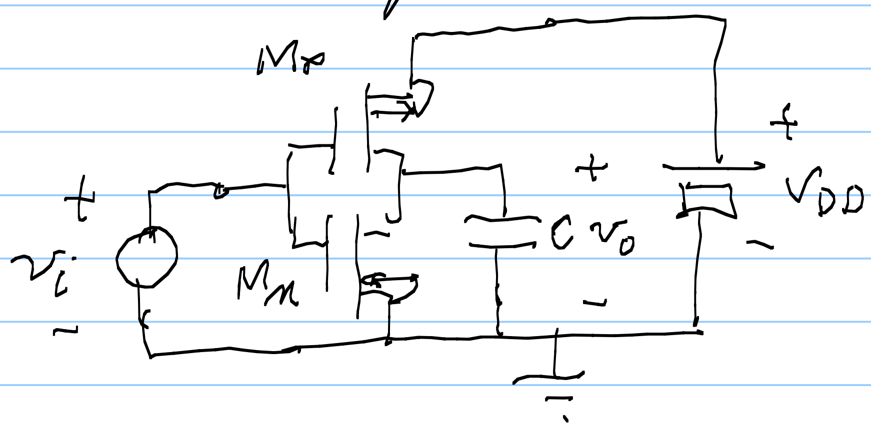
$$S_{in} = 0 + \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} S_{L_2} & 0 \\ 0 & S_{L_3} \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} S_{L_2} & 0 \\ 0 & S_{L_3} \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= 0 + \begin{bmatrix} 0 & S_{L_3} \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ S_{L_2} & 0 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & S_{L_3} \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ -S_{L_2} & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & S_{L_3} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ S_{L_2} & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} S_{L_3} \cdot S_{L_2} & S_{L_3} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = S_{L_3} \cdot S_{L_2}$$ checks before

Riccati eqn.

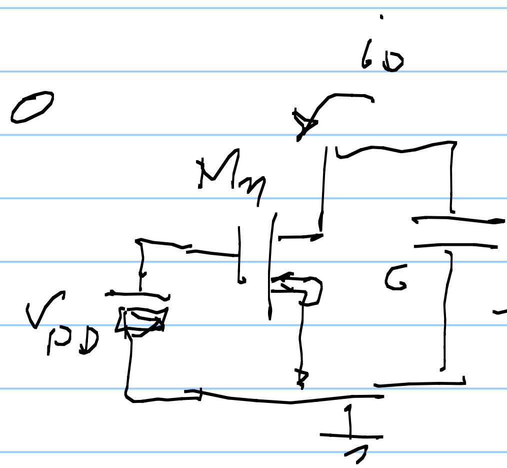


assume $v_i(t) = 0, \; t \le 0$

apply $v_i(t) = V_{DD}, \; t > 0$

$v_o(t) \approx V_{DD}, \quad t \le 0$

find $v_o(t), \; t \geq 0$

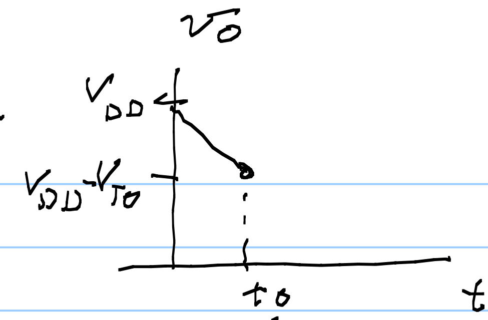


$v_o \Rightarrow v_o(0_+) = V_{DD}$

$V_{DD} - V_{TO} = v_{GS} - V_{TO} < v_{DS} = V_{DD} \Rightarrow M_n$ in saturation

$i_D = k(v_{GS} - V_{TO})^2$

$$C\frac{dv_O}{dt} = -k(V_{DD}-V_{TO})^2$$



when $v_{GS}-V_{TO} \geq v_{DS}$ then switch to Ohmic region (triode)

$$V_{DD}-V_{TO} \geq v_O$$

then $i_D = k\left(2(v_{GS}-V_{TO})v_{DS} - v_{DS}^2\right)$

$$C\frac{dv_O}{dt} = -k\left(2(V_{DD}-V_{TO})v_O - v_O^2\right) \Leftarrow \text{Riccati eq.}$$

$$\frac{C\,dv_O}{k\,v_O\left(2(V_{DD}-V_{TO})-v_O\right)} = dt \qquad t > t_0$$

$$\int_{v_O(t_0)}^{v_O(t)} \frac{dv}{v\left(2(V_{DD}-V_{TO})-v\right)} = -\frac{k}{C}\int_{t_0}^{t} d\tau = -\frac{k}{C}(t-t_0)$$

$$\int \frac{dv}{v(a-v)} = \int \frac{1/a}{v}\,dv + \int \frac{-1/a}{v-a}\,dv \qquad a = 2(V_{00} - V_{\Sigma 0})$$

$$\frac{1}{v(a-v)} = \frac{k_0}{v} + \frac{k_1}{v-a} \Rightarrow k_0 = \frac{1}{a-v}\Big|_{v=0} = 1/a$$

$$k_1 = \frac{v-a}{v(a-v)}\Big|_{v=a} = \frac{-1}{v}\Big|_{v=a} = -1/a$$

$$\int_{v(t_0)}^{v(t)} \frac{\frac{1}{a}\,dv}{v} + \int_{v(t_0)}^{v(t)} \frac{-1/a\,dv}{v-a} = \frac{1}{a}\cdot \ln v\Big]_{v=v_0(t_0)}^{v_0(t)} - \frac{1}{a}\ln(v-a)\Big]_{v=v(t_0)}^{v_0(t)}$$

$$= \frac{1}{a}\ln\frac{v_0(t)}{v_0(t_0)} - \frac{1}{a}\ln\left(\frac{v_0(t)-a}{v_0(t_0)-a}\right)$$

$$= \frac{1}{a}\ln\left(\frac{v_0(t)\cdot(v_0(t_0)-a)}{v_0(t_0)\cdot(v_0(t)-a)}\right) = -\frac{k}{c}(t-t_0)$$

$$\left(\frac{v_o(t) \cdot (v_o(t_0) - a)}{v_o(t_0)\,(v_o(t) - a)} \right) = e^{-\frac{k}{c} a (t - t_0)}$$

$$v_o(t)\left[v_o(t_0) - a\right] = \left[v_o(t_0) \cdot v_o(t) - a\, v_o(t_0)\right] e^{-\frac{k}{c} a (t - t_0)}$$

$$v_o(t)\left[\left(v_o(t_0) - a\right) - v_o(t_0) \cdot e^{-\frac{k}{c} a (t - t_0)}\right] = -a\, v_o(t_0)\, e^{-\frac{ka}{c}(t - t_0)}$$

$$v_o(t) = \frac{-a\, v_o(t_0)\, e^{-\frac{k}{c} a (t - t_0)}}{\left[v_o(t_0) - a\right] - v_o(t_0)\, e^{-\frac{k}{c} a (t - t_0)}}$$

$$a = 2(V_{DD} - V_{T0}), \quad v_o(t_0) = V_{DD} - V_{T0}$$

$$v_o(t) = \frac{-2(V_{DD} - V_{T0})^2\, e^{-\frac{k}{c} a (t - t_0)}}{-(V_{DD} - V_{T0}) - (V_{DD} - V_{T0})\, e^{-\frac{ka}{c}(t - t_0)}}$$

$$v_o(t) = \frac{2(V_{DD}-V_{T0})\, e^{-\frac{ka}{c}(t-t_0)}}{1+e^{-\frac{ka}{c}(t-t_0)}}, \quad t > t_0$$