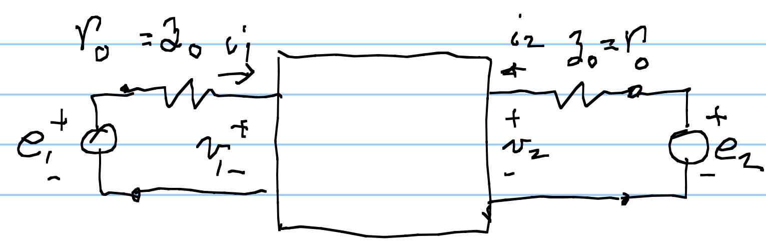
Note Title 10/16/2014

EE610
10/23/14



$$2v^i = v + z_0 i$$

$$2v^r = v - z_0 i$$

$$S v^i = v^r$$

$$v = v^i + v^r$$

$$z_0 i = v^i - v^r$$

$$Y v = i \Rightarrow z_0 Y \cdot v = z_0 i$$

$$z_0 Y (v^i + v^r) = v^i - v^r \Rightarrow \quad (1_n - z_0 Y) v^i = (1_n + z_0 Y) v^r$$

$$S = (1_n + z_0 Y)^{-1} (1_n - z_0 Y) \qquad \hat{S} = (1_n + Y)^{-1} (1_n - Y)$$

BR conditions vs PR $\quad v^{T*}\left[\mathcal{Y}(s) + \mathcal{Y}^{T*}(s)\right] v \geq 0$ for $\sigma > 0$

1st note $\quad S = (1_n + Y)^{-1}(1_n - Y) = (1_n - Y)(1_n + Y)^{-1}$

left $\quad (1_n + Y)\left[(1_n + Y)^{-1}(1_n - Y)\right](1_n + Y) = 1_n - Y + Y - Y^2$

right $\quad (1_n + Y)(1_n - Y)(1_n + Y)^{-1}(1_n + Y) = 1_n + Y - Y - Y^2$

$$(1_n + Y)S = 1_n - Y \Rightarrow S + YS = 1_n - Y \Rightarrow (S - 1_n) = -YS - Y = -Y(1_n + S)$$

$$Y = (1_n - S)(1_n + S)^{-1} = (1_n + S)^{-1}(1_n - S)$$

$$v^{T*}\left[(1_n + S)^{-1}(1_n - S) + (1_n - S^{*T})(1_n + S^{T*})^{-1}\right] v \geq 0 \text{ in } \sigma > 0$$

$$v^{T*}\left[(1_n + S)^{-1}\left\{(1_n - S)(1_n + S^{T*}) + (1_n + S)(1_n - S^{T*})\right\}(1_n + S^{T*})v\right.$$

$$v^{T*}(1_m+S)^{-1}\left\{1_m - \cancel{S} + \cancel{S^{T*}} - SS^{T*} + 1_m + \cancel{S} - \cancel{S^{T*}} - SS^{T*}\right\}(1_m+S^{T*})^{-1}v \geq 0$$

in $\sigma > 0$

$$2\,x^{T*}\left\{1_m - SS^{T*}\right\}x \geq 0 \quad \text{in } \sigma > 0$$

$\Rightarrow \quad 1_m - S(s)S^{T*}(s)$ is positive semi-definite in $\sigma > 0$

(same as $1_m - S^{T*}(s)S(s)$ " " ")

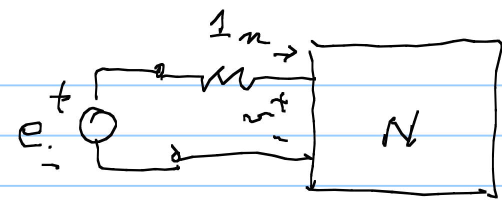
When $s = j\omega \Rightarrow \quad 1_m - S(j\omega)S^T(-j\omega)$

if lossless the above power form $= 0$

then $\quad 1_m = S(j\omega)S^T(-j\omega)$ by analytic continuation

$1_m = S(s)S^T(-s)$ for lossless for all s except

if BR only not true at poles



Ex: $Y(s) = sC$

$S(s) = \dfrac{1-sC}{1+sC}$; $S(-s) = \dfrac{1+sC}{1-sC}$

$S(s)\cdot S(-s) = 1$

$\mathcal{L}_2$ means $\displaystyle\int_{-\infty}^{\infty} e^T(\tau)e(\tau)\,d\tau < \infty$

$$e = v + i \Rightarrow \int_{-\infty}^{\infty} e^T e\,d\tau = \int_{-\infty}^{\infty} \big(\underbrace{v^Tv}_{\geq 0} + \underbrace{i^Ti}_{\geq 0} + \underbrace{2v^Ti}_{\substack{\geq 0 \\ \text{if passive } N}}\big)\,d\tau < \infty$$

$\Rightarrow$ if v^i & e is in $\mathcal{L}_2$ then v & i are in $\mathcal{L}_2$ & v^r is in $\mathcal{L}_2$

but an $\mathcal{L}_2$ map into $\mathcal{L}_2$ always exists $\Rightarrow$ $S(s)$ always exists for a passive N (assumes can apply e in $\mathcal{L}_2$)

$S_{Richards}$: $S_R = \dfrac{1 - y_R}{1 + y_R}$; $y_R(s) = y(k)\,\dfrac{s\,y(s) - k\,y(k)}{s\,y(k) - k\,y(s)}$; $k > 0$

$$= \qquad \frac{y_R}{y(k)} = \frac{\frac{s}{k}\cdot\frac{y(s)}{y(k)} - 1}{\frac{s}{k} - \frac{y(s)}{y(k)}} \qquad \frac{k\,y(k)}{k\,y(k)}$$

$$\hat{y}_R = \frac{\hat{s}\cdot\hat{y} - 1}{\hat{s} - \hat{y}}$$

$$\hat{S}_R = \frac{1 - \hat{y}_R}{1 + \hat{y}_R} = \frac{1 - \frac{\hat{s}\hat{y} - 1}{\hat{s} - \hat{y}}}{1 + \frac{\hat{s}\hat{y} - 1}{\hat{s} - \hat{y}}} = \frac{\hat{s} - \hat{y} - \hat{s}\hat{y} + 1}{\hat{s} - \hat{y} + \hat{s}\hat{y} - 1} = \frac{(1+\hat{s})(1-\hat{y})}{(1-\hat{s})(-1-\hat{y})} = -1\left(\frac{1-\hat{y}}{1+\hat{y}}\right)\left(\frac{1+\hat{s}}{1-\hat{s}}\right)$$

This is BR if y is PR as $\left(\frac{1+\hat{s}}{1-\hat{s}}\right)$ cancel so no pole at $\hat{s} = 1$

$\equiv s = k$ cancels in y_R for rational y

$\Longrightarrow$ if y is PR $\Longrightarrow$ y_R is PR