Note Title — 10/14/2014

EE 610
10/16/14

P. 342 is PR lossless synthesis

Ex: $\frac{V_2}{V_1}(s) = \frac{-k s^2}{s^4 + 2s^3 + 4s^2 + 2s + 2} = \frac{-k \frac{s^2}{2s^3+2s}}{1 + \frac{s^4+4s^2+2}{2s^3+2s}} = \frac{-y_{21}}{1 + y_{22}}$

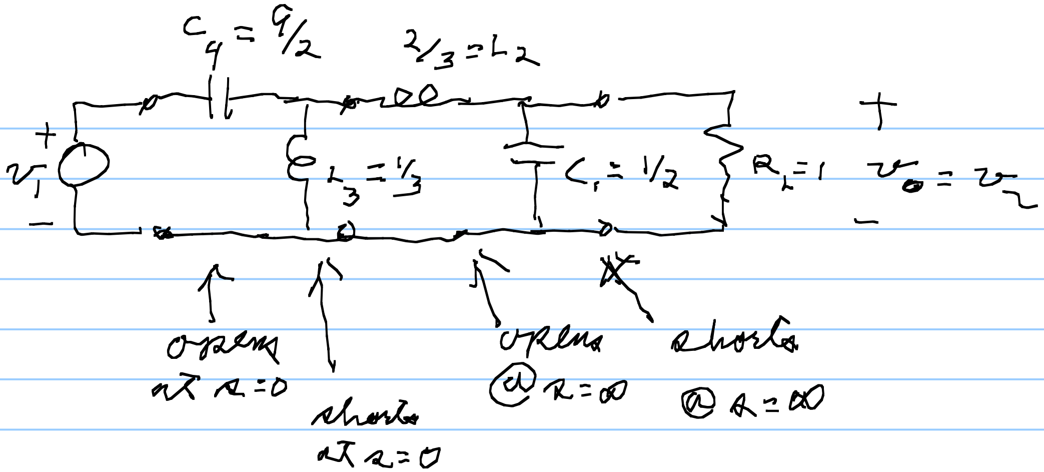
$y_{22} = \frac{s^4 + 4s^2 + 2}{2s^3 + 2s}$ synthesize to get $y_{21} = -k\frac{s^2}{2s^3+2s}$

$$\begin{array}{r|l} & \frac{1}{2}s \\ \hline 2s^3+2s & s^4+4s^2+2 \\ & s^4+s^2 \\ \hline & 3s^2+2 \end{array}$$

$$\begin{array}{r|l} & \frac{2}{3}s \\ \hline 3s^2+2 & 2s^3+2s \\ & 2s^3+\frac{4}{3}s \\ \hline & \frac{2}{3}s \end{array}$$

$y_{22} = \frac{1}{2}s + \frac{3s^2+2}{2s^3+2s}$

$= \underset{\uparrow\, y}{\frac{1}{2}s} + \frac{1}{\underset{\uparrow\, z}{\frac{2}{3}s} + \frac{\frac{2}{3}s}{3s^2+2}} = \underset{\uparrow\, y}{\frac{1}{2}s} + \frac{1}{\underset{\uparrow\, z}{\frac{2}{3}s} + \frac{1}{\underset{\uparrow\, y}{\frac{3}{s}} + \frac{1}{\underset{\uparrow\, z}{\frac{2}{9s}}}}}$

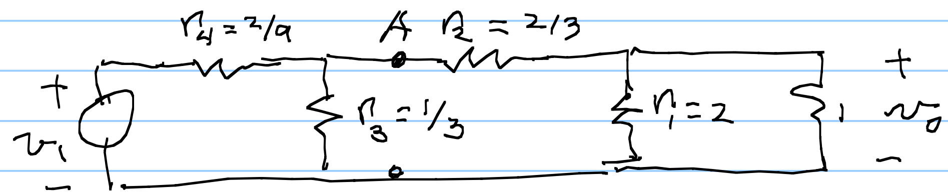


$\frac{2}{3}s$ | $2+3s^2$ | $\frac{3}{s}$

2 | $3s^2$ | $\frac{2}{9s}$

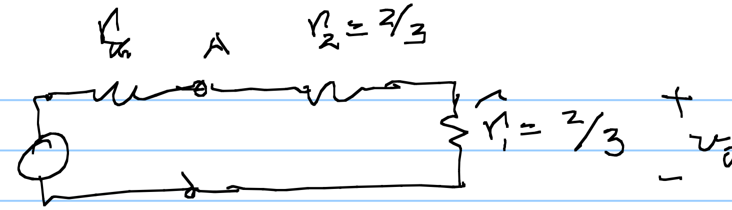
$\frac{2}{3}s$

@ $s=1$, $\frac{v_2}{v_1}(1) = \frac{-k\cdot 1^2}{1^4+2\times1^3+4\times1^2+2\times1+2} = \frac{-k}{11}$



By Thevenin:

$$\hat{v}_1 = \frac{1/3}{\frac{2}{9}+\frac{1}{3}} v_1 = \frac{3}{5} v_1 \Rightarrow$$



$$r_{th} = \frac{1/3 \times 2/9}{1/3 + 2/9} = \frac{2}{15}$$

$$r_2 = \frac{2}{3} + \frac{2}{15} = \frac{2}{15} \times 6 = \frac{4}{5}$$

$\equiv$

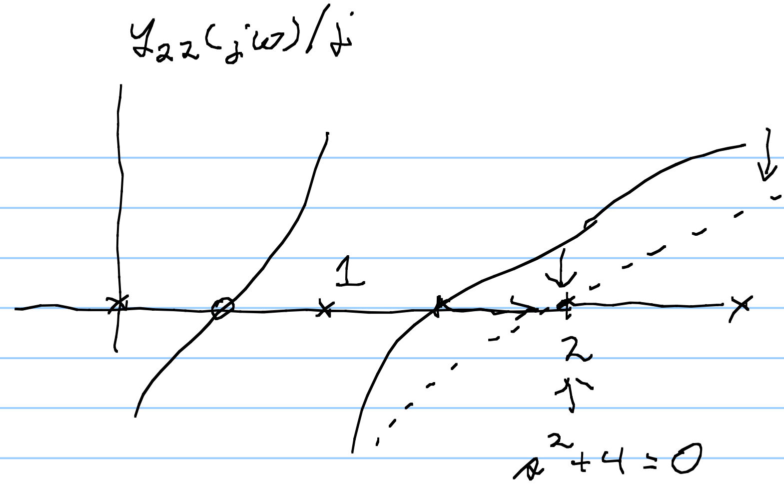
$$v_0 = \frac{2/3}{\frac{2}{3}+\frac{4}{5}} \times \hat{v}_1 = \frac{10}{22} \times \frac{3}{5} v_1 = \frac{6}{22} v_1 = \frac{3}{11} v_1 \Rightarrow \frac{v_0}{v_i} = \frac{3}{11} = \frac{-k}{11}$$

$$-k = 3 \Rightarrow \frac{v_0}{v_i} = \frac{3s^2}{s^4 + 2s^3 + 4s^2 + 2s + 2}$$

Ex:

$$\frac{v_0}{v_i} = \frac{-k(s^2+4)}{s^4 + 2s^3 + 4s^2 + 2s + 2} = \frac{-k\left(\frac{s^2+4}{2s^3+2s}\right)}{1 + \frac{s^4+4s^2+2}{2s^3+2s}} = \frac{-y_{21}}{1+y_{22}}$$

$$2s^3 + 2s = 2s(s^2+1)$$



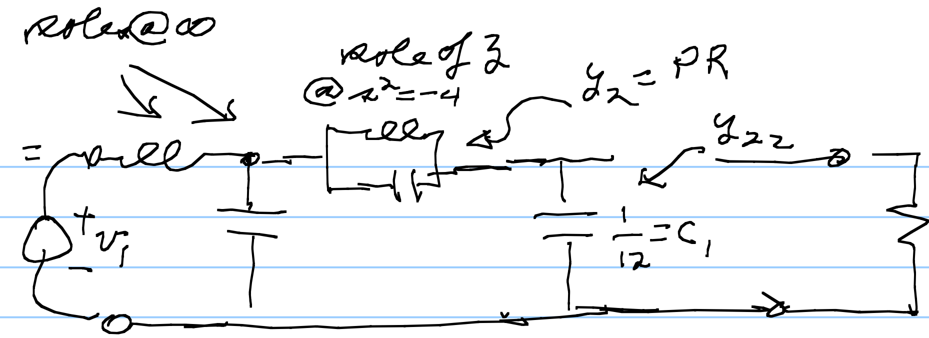
do partial pole removal

$$Y_{22}\Big|_{s=j2} = \frac{(j2)^4+4(j2)^2+2}{2j2((j2)^2+1)} = \frac{16-16+2}{2j2(-3)} = \frac{2}{-j12} = +j\frac{1}{6} = j2\times\frac{1}{12}$$

$\therefore\ Y_{22}(s) - s\cdot\frac{1}{12} = Y_2(s)$ is still PR residue of $Y(s)\Big|_{s=\infty} = \frac{1}{2} > \frac{1}{12}$

$$Y_2 = \frac{s^4+4s^2+2}{2s^3+2s} - \frac{s}{12} = \frac{s^4(1-2/12)+s^2(4-2/12)+2}{2s^3+2s} = \frac{\frac{5}{6}s^4+\frac{23}{6}s^2+2}{2s^3+2s}$$

$$z_2 = \frac{1}{y_2} = \frac{2k_2 s}{s^2+4} + \frac{2k_3 s}{s^2+\omega_2^2} =$$



$$\begin{array}{r|l} & \frac{5}{6}s^2 + \frac{3}{6} \\ \hline s^2+4 & \frac{5}{6}s^4 + \frac{23}{6}s^2 + 2 \\ & \frac{5}{6}s^4 + \frac{20}{6}s^2 \\ \hline & \frac{3}{6}s^2 + 2 \end{array}$$

degree of $\frac{v_0}{v_i} = 4$ but used 5 reactive elements

$$\Rightarrow \frac{5}{6}s^4 + \frac{23}{6}s^2 + 2 = \frac{5}{6}(s^2+4)(s^2+3/5)$$

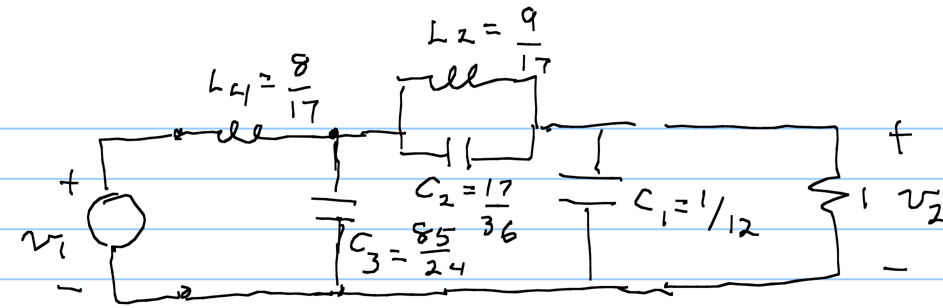
$$z_2 = \frac{2s^3+2s}{\frac{5}{6}(s^2+4)(s^2+3/5)}$$

$$= \frac{\frac{36}{17}s}{s^2+4} + \frac{\frac{24}{85}s}{s^2+3/5}$$

$$2k_2 = \left.\frac{2s^3+2s}{5/6(s^2+3/5)s}\right|_{s^2=-4} = \frac{-2\times4+2}{5/6(-4+3/5)} = \frac{-6\times6}{-20+3} = \frac{36}{17}$$

$$2k_3 = \left.\frac{2s^3+2s}{5/6(s^2+4)s}\right|_{s^2=-3/5} = \frac{-2\times3/5+2}{5/6(-\frac{3}{5}+4)} = \frac{2\times2\times6}{20-3}\times\frac{1}{5} = \frac{24}{85}$$

$$z_2 = \frac{1}{\frac{17}{36}s + \frac{17}{9s}} + \frac{1}{\frac{85}{24}s + \frac{1}{\frac{8}{17}s}} \Rightarrow$$

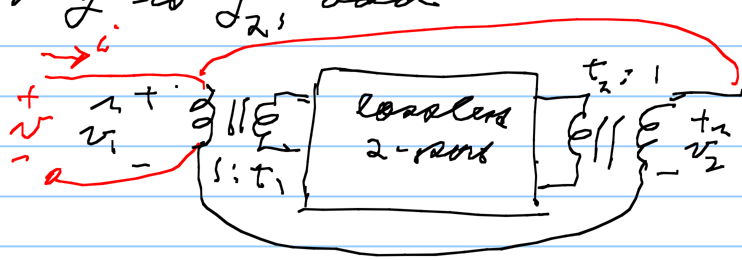


2 zeros of $\frac{v_2}{v_1}$ @ ∞

zero of $\frac{v_2}{v_1}$ @ $s^2 = -4$

partial pole removal

Why do y_{2}'s add?



v_1 / i = lossless PR

desire same voltage & add currents for y

" " current " " voltages for z

$v_1 = t_1 \hat{v}_1$, $v_2 = t_2 \hat{v}_2$, $v = \hat{v}_1 = \hat{v}_2$, $i = +t_1 i_1 + t_2 i_2$

i_1 & i_2 = original 2 port i's

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$+t_1 \cdot i = +t_1 y_{11} v_1 + t_1 y_{12} v_2 = +t_1 y_{11} t_1 \hat{v}_1 + t_1 y_{12} t_2 \hat{v}_2$$

$$+t_2 \cdot i = +t_2 y_{21} t_1 \hat{v}_1 + t_2 y_{22} t_2 \hat{v}_2$$

add & use $\hat{v}_1 = \hat{v}_2 = v$

$Y_{in} = t_1^2 y_{11} + t_1 t_2 y_{12} + t_1 t_2 y_{21} + t_{22}^2 y_{22}$ $\Leftarrow$ PR & lossless

$\Rightarrow$ all poles of y_{12} & y_{21} are in y_{11} & y_{22} & determinant of residue matrix > 0

$$Y_{in} = \begin{bmatrix} t_1 & t_2 \end{bmatrix} \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & t_{22} \end{bmatrix} \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$ $\Leftarrow$ holds at each pole on its residue matrix