Note Title 9/20/2014

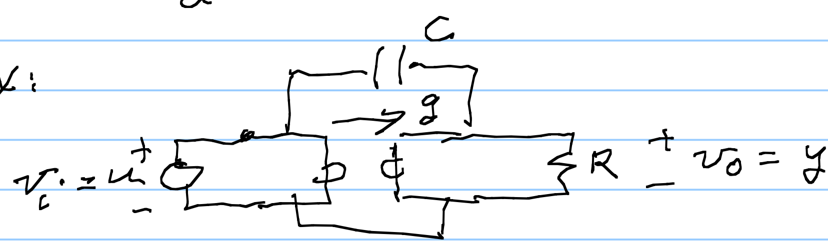
EE 610
09/25/14

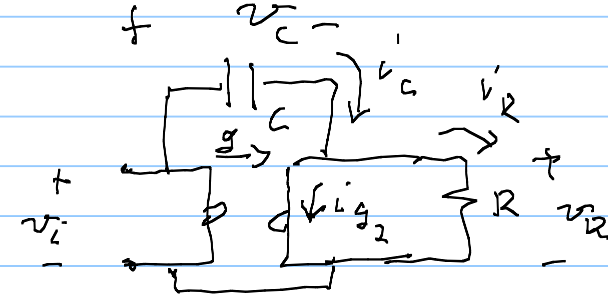
$$E\dot{x} = Ax + Bu, \quad y = Cx$$

state variable equations; $E = 1_k$ $\quad k$ = size of state

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u$$
$$y = \hat{C}\hat{x} + Du + E\dot{u} + E_1\ddot{u} + \cdots$$

Ex:





$$i_{g_2} = -g v_i, \quad i_R = G v_R, \quad G = 1/R$$

$$i_c = C\frac{dv_c}{dt} = i_{g_2} + i_R = -g v_i + G(v_i - v_c)$$

$$C\frac{dv_c}{dt} = -G v_c + (G-g) v_i$$

$$\dot{v}_c = -\frac{G}{C} v_c + \frac{(G-g)}{C} v_i$$

$$y = v_R = R \cdot i_R = v_i - v_c \implies y = -v_c + v_i$$

$$\dot{x} = Ax + Bu \;,\; y = \hat{C}x + Du$$

$$A = -\frac{G}{C} \,,\; \frac{G-g}{C} = B \;,\; \hat{C} = -1 \,,\; D = +1$$

Take $\mathcal{L}[\cdot]$ = Laplace transform

$$sE\,X(s) = A \cdot X(s) + B\,U(s) \implies (sE - A)X(s) = B\,U(s)$$

$$Y(s) = C\,X(s)$$

$$Y(s) = C\left[sE - A\right]^{-1} B\, U(s) = T(s)\cdot U(s)$$

$T(s)$ = transfer function matrix;

if $x \to \hat{x}$ = state then

$$Y(s) = \left\{\hat{C}\left[1_k s - \hat{A}\right]^{-1}\hat{B} + D + \mathcal{E}s + \mathcal{E}_1 s^2 + \dots\right\} U(s)$$

we have

$$E\dot{x} = Ax + Bu \quad , \quad y = Cx$$

Let P = constant nonsingular matrix

$$PE\dot{x} = PAx + PBu \qquad y = Cx$$

Let Q = constant nonsingular, $x = Q\underline{z}$

$$P E Q \dot{\underline{x}} = P A Q \underline{x} + P B u \,, \quad y = C Q \underline{x}$$

$$Y(s) = C Q \left[P E Q s - P A Q \right]^{-1} P B U(s)$$

$$\begin{aligned} T(s) &= C Q \left[P E Q s - P A Q \right]^{-1} P B \\ &= C Q \left[P (E s - A) Q \right]^{-1} P B \\ &= C Q Q^{-1} (E s - A)^{-1} P^{-1} P B = C (E s - A)^{-1} B \end{aligned}$$

Design using state variable eqs.

$$\dot{x} = A x + B u$$

$$y = C x + D u$$

assume $T(s) = Y(s)$ admittance

$u =$ input voltage v_i

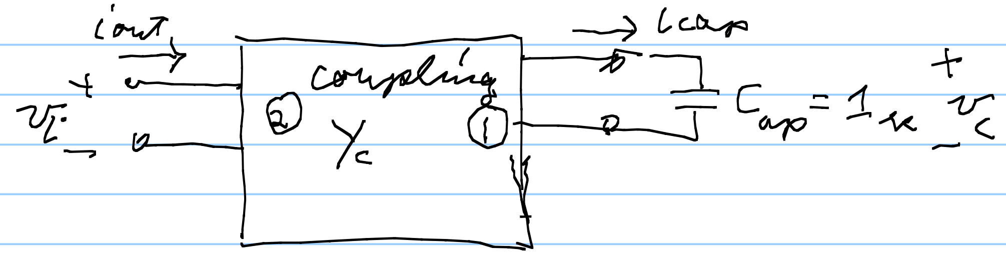
$y = i =$ current

1st assume u & i at <u>same</u> ports

$$C_{cap} = 1_k \Rightarrow \dot{v}_c = A v_c + B v_i = i_{cap}$$

$$y = i_{out} = C v_c + D v_i$$

$$\begin{bmatrix} -i_{cap} \\ i_{out} \end{bmatrix} = \begin{bmatrix} i_{cap} \\ i_{out} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} v_c \\ v_i \end{bmatrix} = \text{constant admittances}$$



$$Y_c = \begin{bmatrix} -A & -B \\ C & D \end{bmatrix} = \text{a constant}, \quad T(a) = C(a 1_k - A)^{-1} B + D$$

Ex: $T(s) = y(s) \Rightarrow$ create $\dot{x} = Ax + Bv_i$, $i_{out} = Cx + Dv_i$

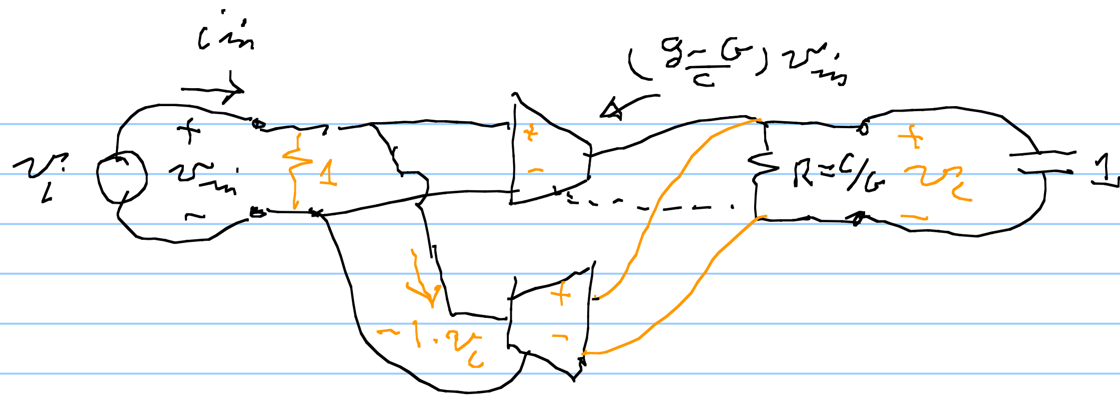
then I can design a circuit by forming

$$Y_c = \begin{bmatrix} -A & -B \\ C & D \end{bmatrix}$$ & loading in unit capacitors

This Y_c is a constant; realize by OTA's of gains $g_{m_{ij}}$ = gain of ijth OTA.

$$\dot{x} = -\frac{G}{C}x + \frac{(G-g)}{C}v_i \qquad \Rightarrow \qquad Y_c = \begin{bmatrix} G/C & \frac{g-G}{C} \\ -1 & +1 \end{bmatrix}$$

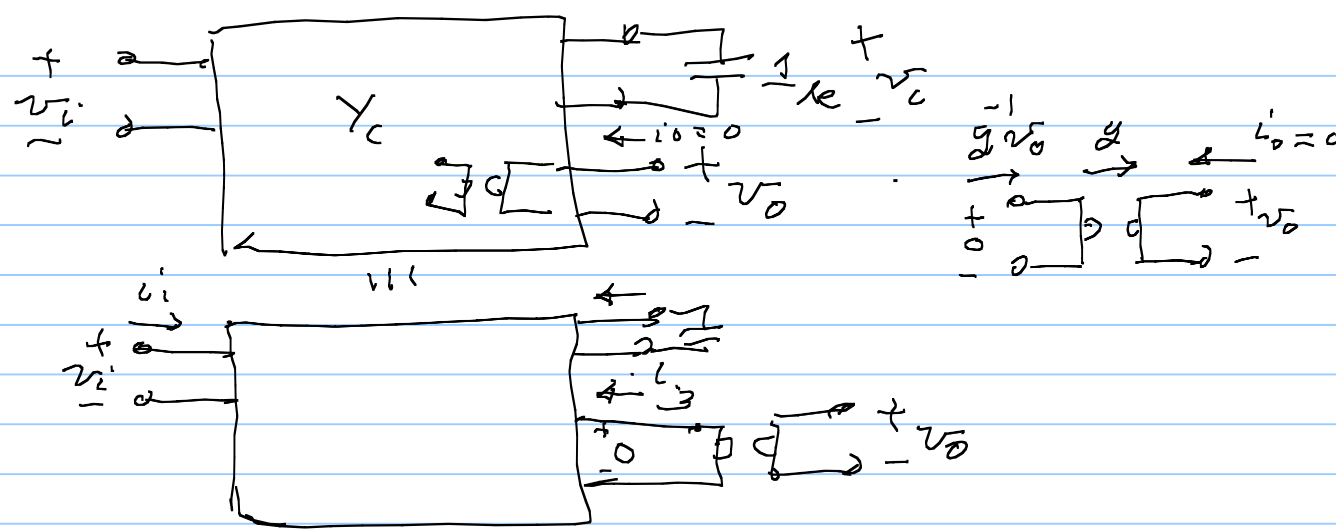
$$i_{in} = -x + v_i$$



If the output is a voltage at another set of ports

$$\dot{x} = Ax + Bv_i$$

$$v_o = Cx + Dv_i$$



$$Y_c \begin{bmatrix} v_c \\ v_i \\ 0 \end{bmatrix} = \begin{bmatrix} -i_c \\ i_{in} \\ i_3 = -g^{-1} v_0 \end{bmatrix} = \begin{bmatrix} -\dot{x} \\ \text{dont care} \\ i_3 = -g^{-1} v_0 \end{bmatrix} = \begin{bmatrix} -A & -B & \cdot \\ \cdot & \cdot & \cdot \\ -g^{-1}C & -g^{-1}D & \cdot \end{bmatrix} \begin{bmatrix} x \\ v_i \\ 0 \end{bmatrix}$$

can fill in dots any way desire; nice to make skew symmetric ($M = -M^T$) or zero