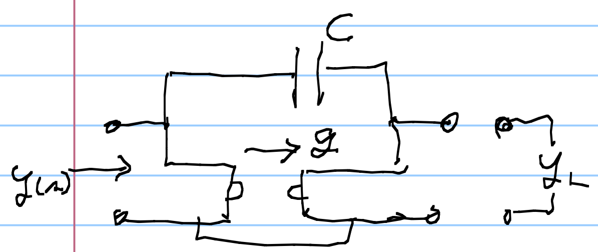
EE 610

09/23/14

Note Title 9/20/2014



$$g^2 = y^2(k)$$

$$C = \frac{y(k)}{k}$$

$$y_L(s) = y(k)\left[\frac{s\,y_i(s) - k\,y_i(k)}{s\,y_i(k) - k\,y_i(s)}\right]$$

Example: $y_i(s) = y(s) = \dfrac{s+1}{s+4}$

$$\mathrm{Ev}\,y(s) = \frac{y(s) + y(-s)}{2}, \quad \mathrm{Od}\,y(s) = \frac{y(s) - y(-s)}{2}$$

$$y(s) = \mathrm{Ev}\,y + \mathrm{Od}\,y \qquad \text{here } 2\,\mathrm{Ev}\,y(s) = \frac{s+1}{s+4} + \frac{(-s)+1}{(-s)+4}$$

$$2\,\mathrm{Ev}\,y(\lambda) = \frac{(\lambda+1)(-\lambda+4)+(\lambda+4)(-\lambda+1)}{(\lambda+4)(-\lambda+4)} = \frac{(-\lambda^2+4\lambda-\lambda+4)+(-\lambda^2-4\lambda+\lambda+4)}{-\lambda^2+16}$$

$$= \frac{-2\lambda^2+8}{-\lambda^2+16} = \frac{-2(\lambda^2-4)}{-\lambda^2+16} = \frac{-2(\lambda-2)(\lambda+2)}{-\lambda^2+16}$$

Choose k = either $+2$ or -2 $\Leftarrow$ zeros of $\mathrm{Ev}\,y(\lambda)$

for the Richards' function

$$y_L(-\lambda) = \frac{-\lambda\, y(-\lambda) - k\, y(k)}{-\lambda\, y(k) - k\, y(-\lambda)}$$

if k is a zero of $\mathrm{Ev}\,y(\lambda) \Rightarrow y(k) = \mathrm{Ev}\,y(k) + \mathrm{Od}\,y(k)$

$$= \mathrm{Od}\,y(k) = -y(-k)$$

implies if $Er\, y(k) = 0$ then $y(-\alpha) = \dfrac{-\alpha\, y(-\alpha) + k\, y(k)}{-\alpha\, y(k) - k\, y(-\alpha)}$

gives also zero over zero at $y(-k)$, $\alpha = -k$ cancels

$\Rightarrow$ $(\alpha + k)(\alpha - k)$ cancel numerator & denominator
$= \alpha^2 - k^2$

$\Rightarrow$ $y_L(\alpha)$ is simpler than $y(\alpha)$

$\therefore$ form $y_L(\alpha) = y(k) \cdot \dfrac{\alpha\, y(\alpha) - k\, y(k)}{\alpha\, y(k) - k\, y(\alpha)}$

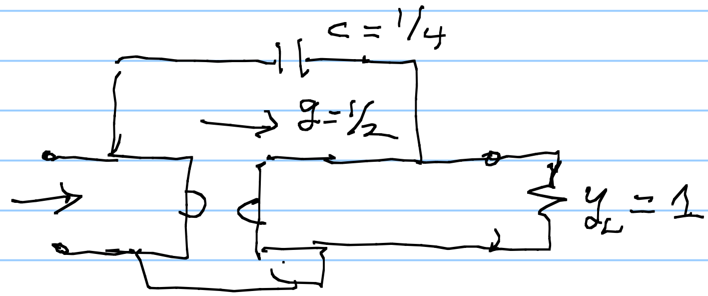
choose $k = +2$, $\quad y_L(\alpha) = \dfrac{k+1}{k+4} \left[\dfrac{\alpha\left(\frac{\alpha+1}{\alpha+4}\right) - 2 \cdot 3/6}{\alpha \cdot 3/6 - 2\left(\frac{\alpha+1}{\alpha+4}\right)} \right] \Rightarrow \dfrac{y_L(\alpha)}{3/6} = \dfrac{\alpha^2 + \cancel{\alpha} - \cancel{\alpha} - 4}{\frac{\alpha^2}{2} + \cancel{2\alpha} - \cancel{2\alpha} - 2}$

$= 2$

$y_L(\alpha) = 1$

$C = \frac{y(k)}{k} = \frac{3/6}{2} = 1/4$, $g^2 = y(k)^2 = (1/2)^2 \Rightarrow g = +1/2$ or $-1/2$

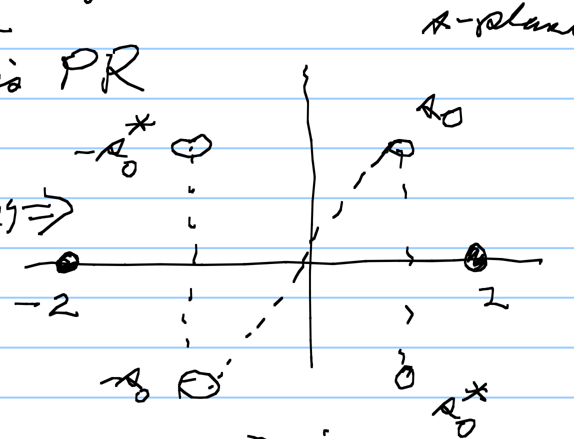
$y(s) = \frac{s+1}{s+4}$



here y_L is PR

if y is PR

for Ev $y(s) \Rightarrow$

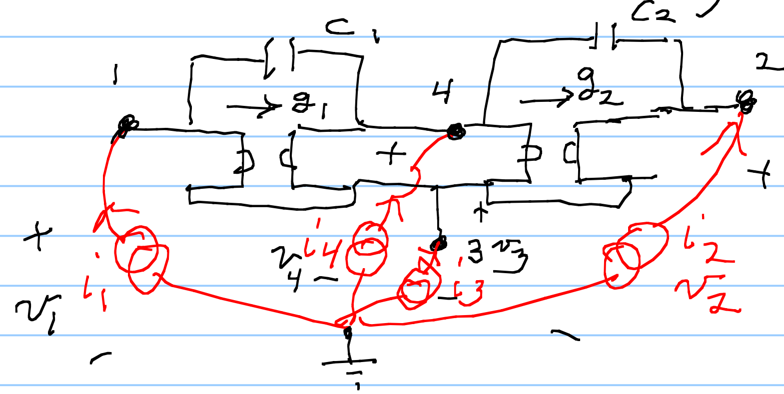


$s = \sigma + j\omega$

$2\,\text{Ev}\,y = y(s) + y(-s) = 0$

$\Rightarrow$ if Ev $y(s) = 0$ then Ev $y(-s) = 0$

indefinite admittance ; $Y_{ind}(s)$
(row entries sum to 0
column entries sum to 0)



$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = Y_{ind}(s) \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$

$$i_1 = sC_1(v_1 - v_4) + g_1(v_4 - v_3)$$

$$i_2 = sC_2(v_2 - v_4) - g_2(v_4 - v_3)$$

$$i_4 = sC_1(v_4 - v_1) - g_1(v_1 - v_3) + sC_2(v_4 - v_2) + g_2(v_2 - v_3)$$

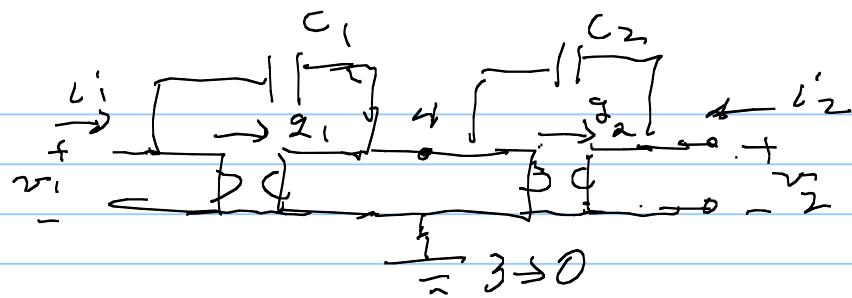
$$Y_{ind} = \begin{bmatrix} sC_1 & 0 & -g_1 & g_1 - sC_1 \\ 0 & sC_2 & g_2 & -sC_2 - g_2 \\ +g_1 & -g_2 & 0 & -g_1 + g_2 \\ -sC_1 - g_1 & -sC_2 + g_2 & g_1 - g_2 & sC_1 + sC_2 \end{bmatrix}$$

ground node 3 $\Rightarrow v_3 = 0 \Rightarrow$ can delete 3rd row

$\therefore$ also delete row 3 as $i_3 = -(i_1 + i_2 + i_4)$

$$Y_{def} = \begin{bmatrix} sC_1 & 0 & g_1 - sC_1 \\ 0 & sC_2 & -sC_2 - g_2 \\ -sC_1 - g_1 & -sC_2 + g_2 & s(C_1 + C_2) \end{bmatrix}$$

if desire the 2-port Y from nodes 1-3 to nodes 2-3



set $i_4 = 0$ to eliminate the internal node 4

$$i_4 = 0 = (-\alpha C_1 - g_1) v_1 + (-\alpha C_2 + g_2) v_2 + \alpha (C_1 + C_2) v_4$$

$$v_4 = \frac{-1}{\alpha(C_1 + C_2)} \left[(-\alpha C_1 - g_1) v_1 + (-\alpha C_2 + g_2) v_2 \right]$$

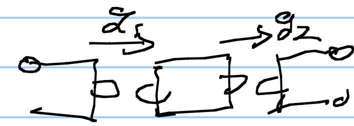
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \alpha C_1 & 0 \\ 0 & \alpha C_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - \frac{1}{\alpha(C_1 + C_2)} \begin{bmatrix} g_1 - \alpha C_1 \\ -\alpha C_2 - g_2 \end{bmatrix} \begin{bmatrix} -\alpha C_1 - g_1 & -\alpha C_2 + g_2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$= \left[Y_{11} - Y_{12} \cdot Y_{22}^{-1} Y_{21} \right] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

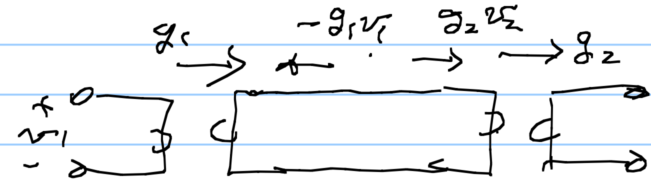
$$\alpha(C_1+C_2)\begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \end{bmatrix} = \begin{bmatrix} \alpha C_1(\alpha C_1+\alpha C_2) - (g_1-\alpha C_1)(-\alpha C_1-g_1) & -(g_1-\alpha C_1)(-\alpha C_2+g_2) \\ 0-(-\alpha C_2-g_2)(-\alpha C_1-g_1) & \alpha C_2(\alpha C_1+\alpha C_2)-(-\alpha C_2-g_2)(-\alpha C_2+g_2) \end{bmatrix} \times \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

lets see what happens

if $C_1 = C_2 = 0 \Rightarrow$ open circuits



$$\frac{\dot{i}_1}{\dot{i}_2} = \frac{[0 - g_1(-g_1)v_1 - g_1 g_2 v_2]}{[-g_1 g_2 v_1 + g_2^2 v_2]} = \frac{g_1}{g_2}\frac{(g_1 v_1 - g_2 v_2)}{(-g_1 v_1 + g_2 v_2)} = -\frac{g_1}{g_2}$$



$\Rightarrow -g_1 v_1 = -g_2 v_2$ by KCL

$$\frac{v_2}{v_1} = \frac{g_1}{g_2}$$

$v_2 = \frac{g_1}{g_2} v_1 \quad g_2 \dot{i}_1 + g_1 \dot{i}_2 = 0 \Rightarrow$ ideal Transformer (no γ)