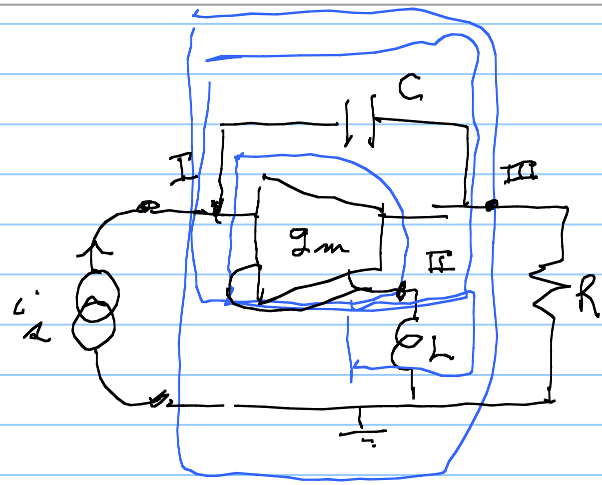
Note Title 9/15/2014

EE 610
09/18/14

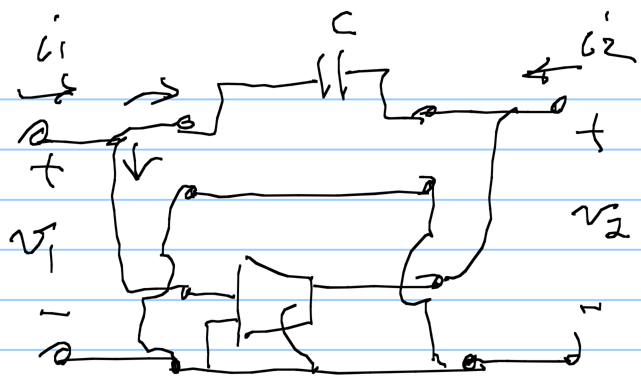


Richards' function, p. 361

$$Z_R(s) = \frac{k Z(s) - s Z(k)}{k Z(k) - s Z(s)}$$

$$i = Y v$$

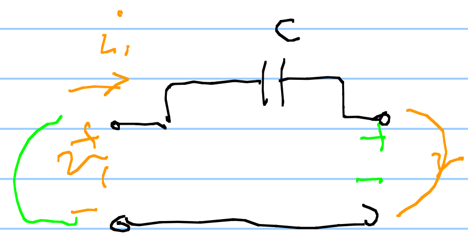
for 2-port
$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$



$$Y_{sub} = \begin{bmatrix} sC & -sC \\ -sC & sC \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ g_m & 0 \end{bmatrix} = \begin{bmatrix} sC & -sC \\ g_m - sC & sC \end{bmatrix}$$

$$\begin{array}{ccc} & \text{C} & \text{OTA} \end{array}$$

$$\Delta_{y_{sub}} = (sC)^2 - (-sC)(g_m - sC)$$
$$= g_m C s$$



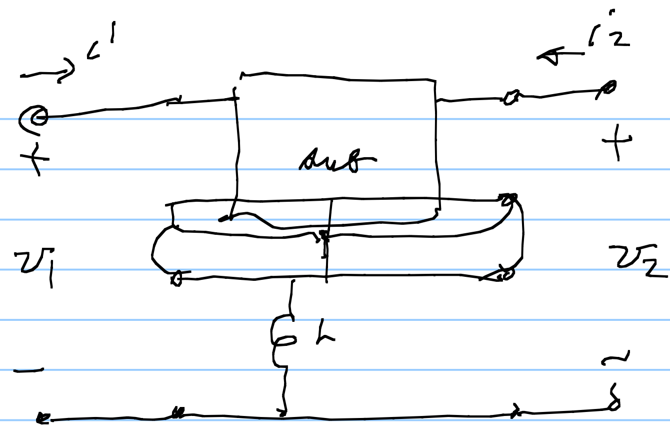
$y_{11} = \dfrac{i_1}{v_1}\Big|_{v_2=0}$ short $= sC$

$y_{12} = \dfrac{i_1}{v_2}\Big|_{v_1=0} = -sC$

By $2 \Leftrightarrow 1$

$y_{22} = sC$

$y_{21} = -sC$

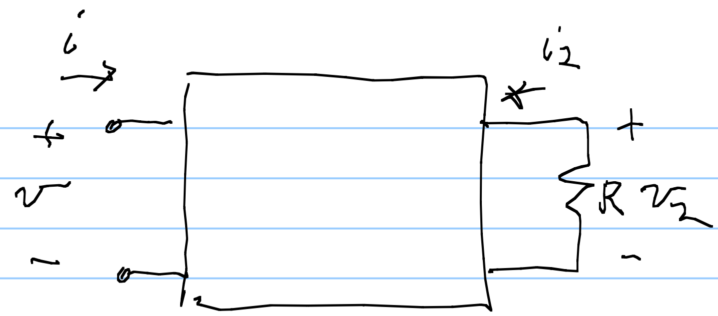


$$Z = Z_{sub} + Z_L$$

$$v = Zi$$
$$i = Yv$$
$$\to Y^{-1} i = Y^{-1} Y v = v$$
$$Z = Y^{-1}$$

$$Z_{sub} = \frac{1}{\Delta_y} \begin{bmatrix} \Delta_{11} & \Delta_{21} \\ \Delta_{12} & \Delta_{22} \end{bmatrix} = \frac{1}{g_m C \alpha} \begin{bmatrix} \alpha C & \alpha C \\ \alpha C - g_m & \alpha C \end{bmatrix}, \quad Z_L = \begin{bmatrix} \alpha L & \alpha L \\ \alpha L & \alpha L \end{bmatrix} \leftarrow \text{by symmetry}$$

$$Z = \begin{bmatrix} \frac{1}{g_m} + \alpha L & \frac{1}{g_m} + \alpha L \\ \frac{1}{g_m} - \frac{1}{C\alpha} + \alpha L & \frac{1}{g_m} + \alpha L \end{bmatrix}; \quad \Delta_z = \frac{1}{C\alpha}\left(\frac{1}{g_m} + \alpha L\right)$$



$$\begin{bmatrix} v_1 = v \\ v_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} i_1 = i \\ i_2 \end{bmatrix}, \quad i_2 = -G v_2$$

i_1 $\quad v_2 = z_{21} i_1 + z_{22}(-G) v_2 \Rightarrow (1 + G z_{22}) v_2 = z_{21} i$

$$v = z_{11} i + z_{12}(-G) v_2 = z_{11} \cdot i - \frac{G z_{12} z_{21}}{1 + G z_{22}} \cdot i =$$

$$= \left[z_{11} - \frac{G z_{12} z_{21}}{1 + G z_{22}} \right] = \left[\frac{z_{11} + G(z_{11} z_{22} - z_{12} z_{21})}{1 + G z_{22}} \right] i$$

$$z_{in} = \frac{R z_{11} + \Delta_z}{R + z_{22}} \; ; \quad y_{in} = \frac{1}{z_{in}} = \frac{R + z_{22}}{R z_{11} + \Delta_z}$$

$$y_{in}(s) = \frac{R + \left(\frac{1}{g_m} + sL\right)}{R\left(\frac{1}{g_m} + sL\right) + \frac{1}{Cs}\left(\frac{1}{g_m} + sL\right)} = \frac{Cs\left(R + \frac{1}{g_m} + sL\right)}{R \cdot \frac{1}{g_m} Cs + s^2 LC + \frac{1}{g_m} + sL}$$

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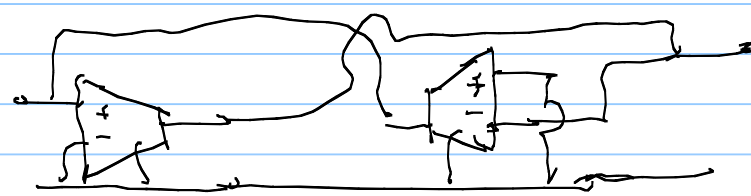
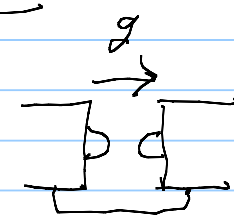
gyrator



$$Y_{gyr} \approx \begin{bmatrix} 0 & -g_m \\ g_m & 0 \end{bmatrix} \Longleftrightarrow \underset{\text{gyrator}}{} $$

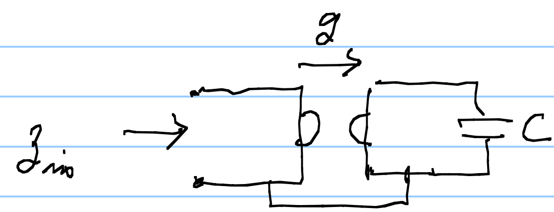


$$Y = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix}$$

$$\det Y = g^2 \Longrightarrow Z_{gyr} = Y^{-1} = \frac{1}{g^2}\begin{bmatrix} 0 & -g \\ g & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1/g \\ 1/g & 0 \end{bmatrix} = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix}$$

g = gyration conductance, r = gyration resistance

$$P_{in}(t) = v_1 i_1 + v_2 i_2 = v_1 \cdot g v_2 + v_2(-g v_1) = v_1 v_2 (g-g) \equiv 0$$

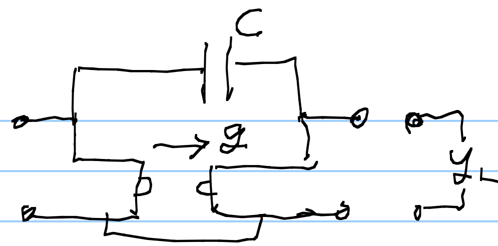


$\Rightarrow$

$$v = (-r)\, i_2 = -r(-sC)\, v_2$$

$$v_2 = r \cdot i_1 = r i \Rightarrow i = \frac{v_2}{r}$$

$$Z_{in} = \frac{v}{i} = \frac{r s C\, v_2}{v_2/r} = r^2 s C \equiv sL, \quad L = r^2 C$$



$$Y = \begin{bmatrix} sC & -sC+g \\ -sC-g & sC \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Y \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} i_1 \\ -y_L v_2 \end{bmatrix} \Rightarrow \quad i_2 = y_{21} v_1 + y_{22} v_2 = -y_L v_2$$

$$(-y_L - y_{22}) v_2 = y_{21} v_1$$

$$i_1 = i = y_{11} v_1 + y_{12} v_2 = y_{11} v_1 - \frac{y_{12} \cdot y_{21}}{y_{22} + y_L} v_1$$

$$y_i = Y_{in} = \frac{i_1}{v_1} = \frac{\Delta y + y_{11} y_L}{y_{22} + y_L} = \frac{g^2 + sC\, y_L}{sC + y_L}$$

$$y_i\cdot(\alpha C + y_L) = g^2 + \alpha C\, y_L \Rightarrow y_i\,\alpha C + y_i\cdot y_L = g^2 + \alpha C\, y_L$$

$$(\alpha C - y_i)\,y_L = \alpha C\, y_i - g^2$$

$$y_L = \frac{\alpha C\, y_i - g^2}{\alpha C - y_i} = \frac{\frac{\alpha C}{g^2}\cdot y_i - 1}{\alpha - y_i/C}\cdot\frac{g^2}{C}$$

$$y_R = \frac{\alpha\, y(\alpha) - k\, y(k)}{\alpha\, y(k) - k\cdot y(\alpha)} \Leftarrow \text{Richards' function}$$

$$= \frac{k\, y(k)}{y(k)}\left\{\frac{\left[\frac{\alpha}{k\, y(k)}\cdot y(\alpha) - 1\right]}{\left[\alpha - \frac{k}{y(k)}\cdot y(\alpha)\right]}\right\} \Rightarrow \left.\begin{array}{l}\frac{C}{g^2} = \frac{1}{k\, y(k)} \\ \frac{1}{C} = \frac{k}{y(k)}\end{array}\right\} \text{identify}$$

given k & $y(k)$ then $\frac{c}{g^2} = \frac{c}{1} \cdot \frac{1}{g^2} = \frac{y(k)}{k} \cdot \frac{1}{g^2} = \frac{1}{k\,y(k)}$

$$\Rightarrow g^2 = y^2(k) \Rightarrow g = y(k)$$

$$c = \frac{y(k)}{k}$$

$$y_L = \frac{g^2}{c}\left[\frac{\frac{\alpha}{k\,y(k)}\,y(\alpha) - 1}{\alpha - \frac{k}{y(k)} \cdot y(\alpha)}\right] \qquad y_R = k\left[\frac{\frac{\alpha}{k\,y(k)}\,y(\alpha) - 1}{\alpha - \frac{k}{y(k)}\,y(\alpha)}\right]$$

$$\frac{y^2(k)}{\frac{y(k)}{k}} = k\,y(k) \qquad y_R(\alpha) \times y(k) = y_L(\alpha)$$

$$y_L(\alpha) = y(k)\left[\frac{\alpha\, y_i(\alpha) - k\, y_i(k)}{\alpha\, y_i(k) - k\, y_i(\alpha)}\right]$$