## Problem 1

a) Synthesize $y(s) = \dfrac{5s^3 + 21s}{s^4 + 8s^2 + 15}$

### 1st Foster

$$Z(s) = \frac{(s^2+3)(s^2+5)}{s(5s^2+21)} = \frac{k_0}{s} + k_\infty s + \frac{2k_2 s}{5s^2+21}$$

$$Z(s)\, s(5s^2+21) = k_0(5s^2+21) + k_\infty s^2(5s^2+21) + 2k_2 s^2 = (s^2+3)(s^2+5) = F(s)$$

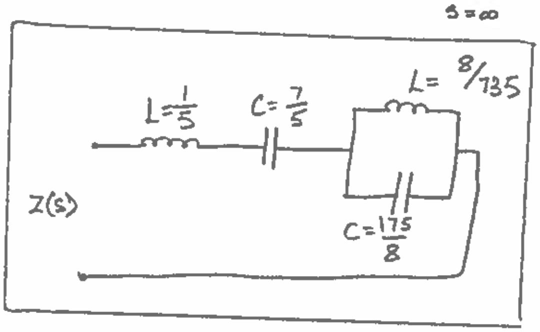
$$F(s)\Big|_{s=0} \Rightarrow 21k_0 = 15 \Rightarrow k_0 = \frac{5}{7}$$

$$F(s)\Big|_{s^2=\frac{-21}{5}} \Rightarrow 2k_2\left(\frac{-21}{5}\right) = \frac{-24}{25} \Rightarrow 2k_2 = \frac{8}{35}$$

$$k_\infty = \frac{(s^2+3)(s^2+5) - 2k_2 s^2 - k_0(5s^2+21)}{s^2(5s^2+21)}\Bigg|_{s=\infty} \Rightarrow k_\infty = \frac{1}{5}$$

$$Z(s) = \frac{5}{7s} + \frac{1}{5}s + \frac{\frac{8}{35}s}{5s^2+21} = \frac{5}{7s} + \frac{1}{5}s + \frac{1}{\frac{175}{8}s + \frac{735}{8s}}$$

$\Rightarrow$




### 2nd Foster.

$$y(s) = \frac{s(5s^2+21s)}{(s^2+3)(s^2+5)} = \frac{k_0}{s} + k_\infty s + \frac{2k_2 s}{s^2+3} + \frac{2k_4 s}{s^2+5}$$

$$y(s)(s^2+3)(s^2+5)\cdot s = k_0(s^2+3)(s^2+5) + k_\infty s^2(s^2+3)(s^2+5) + 2k_2 s^2(s^2+5) + 2k_4 s^2(s^2+3) = s^2(5s^2+21) = F(s)$$

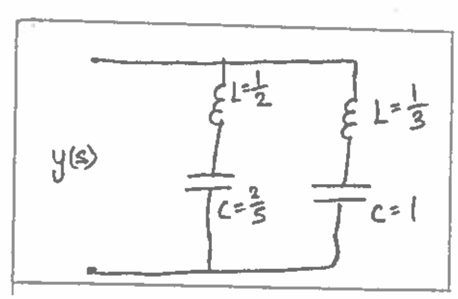
$$F(s)\Big|_{s=0} \Rightarrow k_0 = 0$$

$$\text{Solve for } k_\infty\Big|_{s=\infty} \Rightarrow k_\infty = 0$$

$$F(s)\Big|_{s^2=-5} \Rightarrow 2k_4 = 2$$

$$F(s)\Big|_{s^2=-3} \Rightarrow 2k_2 = 3$$

$$y(s) = \frac{3s}{s^2+3} + \frac{2s}{s^2+5} = \frac{1}{\frac{1}{3}s + \frac{1}{s}} + \frac{1}{\frac{1}{2}s + \frac{5}{2s}}$$



## $1^{st}$ Cauer

$$y(s) = \frac{5s^3+21s}{s^4+8s^2+15} \qquad Z(s) = \frac{s^4+8s^2+15}{5s^3+21s}$$

$\frac{1}{5}s$

$5s^3+21s \,\big)\, s^4+8s^2+15$

$s^4+\frac{21}{5}s^2$

$\frac{25}{19}s$

$\frac{19}{5}s^2+15 \,\big)\, 5s^3+21s$

$5s^3+\frac{375}{19}s$

$\frac{361}{120}s$

$\frac{24}{19}s \,\big)\, \frac{19}{5}s^2+15$

$\frac{19}{5}s^2$

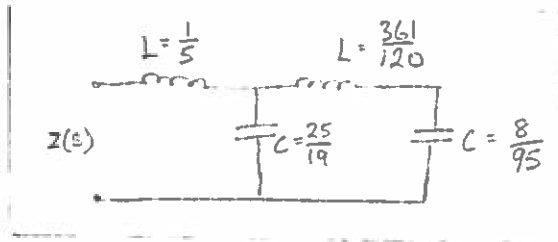
$\frac{8}{95}s$

$15 \,\big)\, \frac{24}{19}s$

$\frac{24}{19}s$

$0$

$$Z(s) = \frac{1}{5}s + \cfrac{1}{\frac{25}{19}s + \cfrac{1}{\frac{361}{120}s + \cfrac{1}{\frac{8}{95}s}}}$$




## $2^{nd}$ Cauer

$$Z(s) = \frac{15+8s^2+s^4}{21s+5s^3}$$

$\frac{15}{21s}$

$21s+5s^3 \,\big)\, 15+8s^2+s^4$

$15+\frac{75}{21}s^2$

$\frac{147}{31s}$

$\frac{31}{7}s^2+s^4 \,\big)\, 21s+5s^3$

$21s+\frac{147}{31}s^3$

$\frac{961}{56s}$

$\frac{8}{31}s^3 \,\big)\, \frac{31}{7}s^2+s^4$

$\frac{31}{7}s^2$

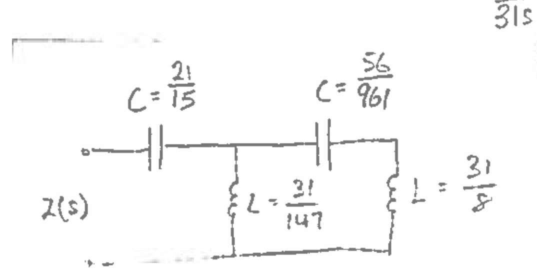
$\frac{8}{31s}$

$s^4 \,\big)\, \frac{8}{31}s^3$

$\frac{8}{31}s^3$

$0$

$$Z(s) = \frac{15}{21s} + \cfrac{1}{\frac{147}{31s} + \cfrac{1}{\frac{961}{56s} + \cfrac{1}{\frac{8}{31s}}}}$$



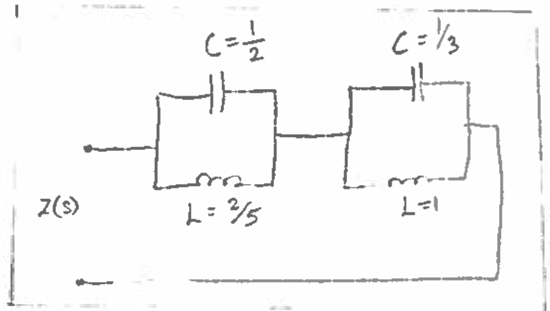
b) Synthesize $Z(s) = \dfrac{5s^3 + 21s}{s^4 + 8s^2 + 15}$

<u>1st Foster</u>

Since $z(s)$ of (b) is equal to $y(s)$ of (a), we can use previously derived result

$\Rightarrow$ from 2nd Foster of (a):

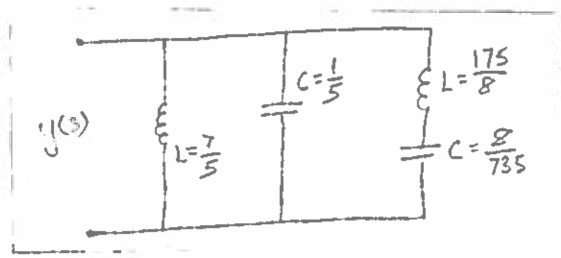
$$Z(s) = \frac{1}{\frac{1}{3}s + \frac{1}{s}} + \frac{1}{\frac{1}{2}s + \frac{5}{2s}}$$




<u>2nd Foster</u>

$\Rightarrow$ from 1st Foster of (a):

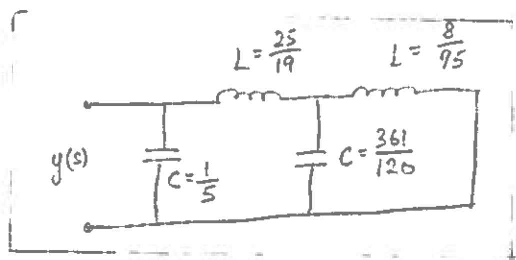
$$y(s) = \frac{5}{7s} + \frac{1}{5}s + \frac{1}{\frac{175}{8}s + \frac{735}{8s}}$$




<u>1st Cauer</u>

$\Rightarrow$ from 1st Cauer of (a):

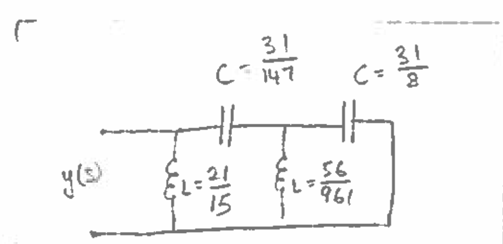
$$y(s) = \frac{1}{5}s + \cfrac{1}{\frac{25}{19}s + \cfrac{1}{\frac{361}{120}s + \cfrac{1}{\frac{8}{95}s}}}$$




<u>2nd Cauer</u>

$\Rightarrow$ from 2nd Cauer of (a):

$$y(s) = \frac{15}{21s} + \cfrac{1}{\frac{147}{31s} + \cfrac{1}{\frac{961}{56s} + \cfrac{1}{\frac{8}{31s}}}}$$



## Comparison of (a), (b)

Since $Z(s)$ of (a) and (b) are inverses of each other, the forms of the synthesized circuits are similar. $C$ and $L$ values are interchanged between the two admittances and series/parallel connections have been inverted as well.

c) Synthesize $y(s) = \dfrac{5s}{s^2+7}$ using Richards's extractions at $k = 1, -1$

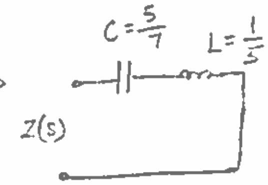
$$y_R(s) = \frac{s\,y(s) - k\,y(k)}{s\,y(k) - k\,y(s)} \Rightarrow Z_R(s) = \frac{k\,z(s) - s\,z(k)}{k\,z(k) - s\,z(s)} \Rightarrow Z(s) = \frac{1}{\dfrac{1}{Z_R(s)Z(k)} + \dfrac{s}{k\,z(k)}} + \frac{1}{\dfrac{k}{s\,z(k)} + \dfrac{Z_R(s)}{Z(k)}}$$

[Richards's in y] [Richards's in z] [Solve for Z(s)]

$k=1$

$$Z(s) = \frac{s^2+7}{5s} \qquad z(s=k=1) = \frac{8}{5} \Rightarrow Z_R(s) = \frac{\dfrac{s^2+7}{5s} - s\dfrac{8}{5}}{\dfrac{8}{5} - \dfrac{s(s^2+7)}{5s}} = \frac{7}{s}$$

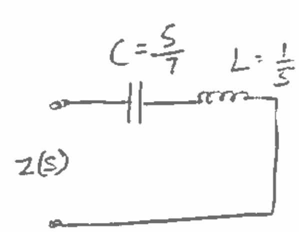
$$\therefore Z(s) = \frac{1}{\dfrac{1}{\left(\frac{7}{s}\right)\frac{8}{5}} + \dfrac{s}{\frac{8}{5}}} + \frac{1}{\dfrac{1}{s\frac{8}{5}} + \dfrac{7/s}{8/5}} = \frac{7}{5s} + \frac{1}{5}s \Rightarrow$$




d) $k = -1$

$$Z(s=k=-1) = \frac{-8}{5} \qquad Z_R(s) = \frac{-\dfrac{s^2+7}{5s} + s\dfrac{8}{5}}{\dfrac{8}{5} - \dfrac{s(s^2+7)}{5s}} = \frac{-7}{s}$$

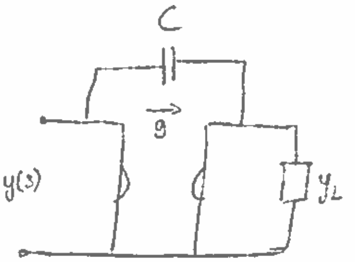
$$\therefore Z(s) = \frac{1}{\dfrac{1}{\left(\frac{-7}{s}\right)\left(\frac{-8}{5}\right)} + \dfrac{s}{(-1)\left(\frac{-8}{5}\right)}} + \frac{1}{\dfrac{-1}{-s\frac{8}{5}} + \dfrac{-7/s}{-8/5}} = \frac{7}{5s} + \frac{1}{5}s$$




Extractions at $k=1$ and $k=-1$ yield the same result

Alternative for 1c, 1d

c) $y(s) = \dfrac{5s}{s^2+7}$




$C = \dfrac{y(k)}{k}$ $\quad g^2 = y(k)^2$

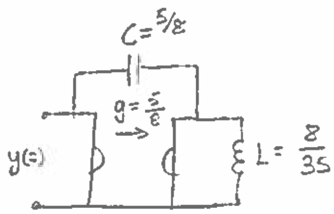
$k=1$

$$y_L = y(k)\left[\frac{s\,y(s) - k\,y(k)}{s\,y(k) - k\,y(s)}\right] = \frac{5}{8}\,\frac{s\frac{5s}{s^2+7} - \frac{5}{8}}{s\frac{5}{8} - \frac{5s}{s^2+7}} = \frac{35}{8s} \Rightarrow Z_L = \frac{8}{35}s = sL$$

$y(k=1) = \dfrac{5}{8}$

$g = \pm\dfrac{5}{8}$

$\Rightarrow$



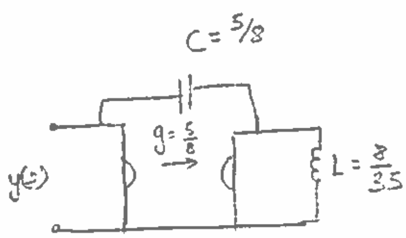

d) $k=-1$

$y(k=-1) = \dfrac{-5}{8}$

$y_L = \dfrac{35}{8s}$

$g = \pm\dfrac{5}{8}$

$\Rightarrow$




• Extractions at $k=1$ and $k=-1$ yield the same result

# Problem 2

a) Synthesize transfer function of a lossless ladder 2-port

$$y_{21}(s) = \frac{V_2}{V_1} = \frac{k}{s^3 + 2s^2 + 2s + 1}$$

- Numerator is even $\therefore$ divide out odd part of denominator:

$$\frac{V_2}{V_1} = \frac{\dfrac{k}{s^3+2s}}{1 + \dfrac{2s^2+1}{s^3+2s}} = \frac{-y_{21}}{1 + y_{22}}$$

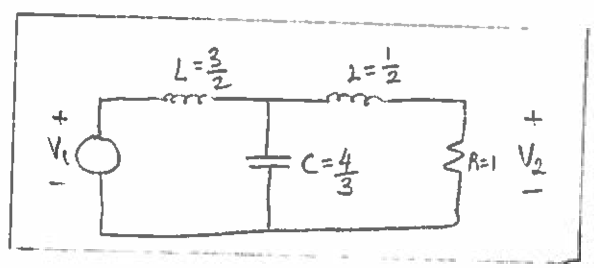
- Synthesize $y_{22}$:

$$Z_{22} = \frac{s^3+2s}{2s^2+1}$$

$$\begin{array}{r|l} 2s^2+1 & s^3+2s \quad \to \tfrac{1}{2}s \\ & s^3+\tfrac{1}{2}s \\ \hline \tfrac{3}{2}s & 2s^2+1 \quad \to \tfrac{4}{3}s \\ & 2s^2 \\ \hline 1 & \tfrac{3}{2}s \quad \to \tfrac{3}{2}s \\ & \tfrac{3}{2}s \\ \hline & 0 \end{array}$$

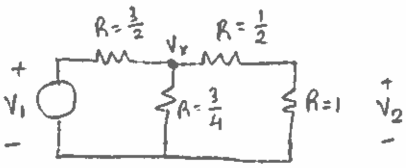
$$\therefore \; Z_{22}(s) = \frac{1}{2}s + \cfrac{1}{\frac{4}{3}s + \cfrac{1}{\frac{3}{2}s}}$$

$\Rightarrow$




b) At $s=1$

$$\frac{V_2}{V_1}(s=1) = \frac{k}{1+2+2+1} = \frac{k}{6}$$




Solve for $V_2/V_1$

$$\frac{V_2}{V_x} = \frac{1}{1+\frac{1}{2}} = \frac{2}{3} \qquad R_L = \left(1+\tfrac{1}{2}\right) \parallel \left(\tfrac{3}{4}\right) = \frac{\frac{3}{2}\times\frac{3}{4}}{\frac{3}{2}+\frac{3}{4}} = \frac{1}{2}$$

$$\frac{V_x}{V_1} = \frac{R_L}{R_L+\frac{3}{2}} = \frac{1/2}{1/2+\frac{3}{2}} = \frac{1}{4}$$

$$\frac{V_2}{V_1} = \frac{V_2}{V_x}\times\frac{V_x}{V_1} = \frac{2}{3}\times\frac{1}{4} = \frac{1}{6} \implies \frac{1}{6} = \frac{k}{6} \quad \therefore \boxed{k=1}$$