ENEE 610

Homework 5

Problem 1

$$\frac{d^3z}{dt^3} + 5\frac{d^2z}{dt^2} + 6\frac{dz}{dt} + 7z = 3v \qquad i = 2v - 3\frac{dz}{dt}$$

i)

Let $z_1 = z$

$z_2 = \dot{z}_1 = \dot{z}_0$

$z_3 = \dot{z}_2 = \ddot{z}_1 = \ddot{z}_0$

$z_4 = \dot{z}_3 = \ddot{z}_2 = \dddot{z}_1 = \dddot{z}_0 = -5\ddot{z}_0 - 6\dot{z}_0 - 7z_0 + 3v$

$$\Rightarrow \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & -6 & -5 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} v$$

$$i = \begin{bmatrix} 0 & -3 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + 2v$$

b) The state variable equations are of the form:

$$\left.\begin{aligned} \dot{z} &= Az + Bu \\ y &= Cz + Du \end{aligned}\right\} \quad \begin{aligned} y &= i(s) = \text{output} \\ u &= V(s) = \text{input} \end{aligned}$$

$$\therefore \frac{i(s)}{V(s)} = \frac{y}{U} = T(s) = C(sI_3 - A)^{-1}B + D$$

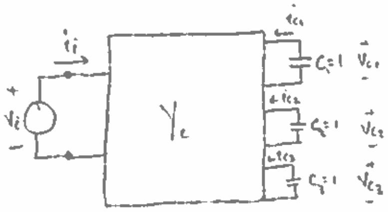
$$= \begin{bmatrix} 0 & -3 & 0 \end{bmatrix} \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 7 & 6 & 5 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} + 2$$

$$\frac{i(s)}{V(s)} = T(s) = \frac{2s^3 + 10s^2 + 3s + 14}{s^3 + 5s^2 + 6s + 7}$$

Zeros:

$2s^3 + 10s^2 + 3s^2 + 14 = 0 \Rightarrow$ $s = -4.981,\ -0.095 + 1.1854i,\ -0.095 - 1.1854i$

c) Constant coupling admittance matrix



$$\begin{bmatrix} i_{c1} \\ i_{c2} \\ i_{c3} \\ i_i \end{bmatrix} = Y_c \begin{bmatrix} v_{c1} \\ v_{c2} \\ v_{c3} \\ v_i \end{bmatrix} \Rightarrow \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ i_i \end{bmatrix} = \begin{bmatrix} -A & -B \\ C & D \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ v_i \end{bmatrix}$$

$$\therefore \quad Y_c = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 7 & 6 & 5 & -3 \\ 0 & -3 & 0 & 2 \end{bmatrix}$$

Problem 2

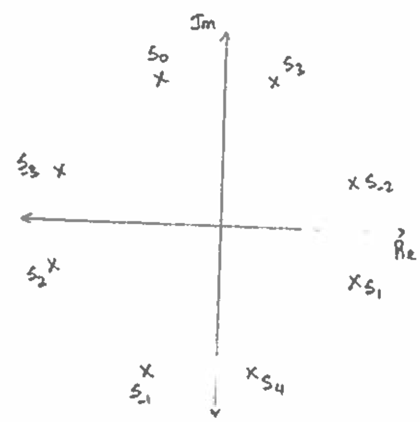
a) The degree 4 transfer function will be of the form:

$$T(s) = \frac{A}{(s-s_a)(s-s_b)(s-s_c)(s-s_d)}$$

Poles are found as:

$$s = e^{j\left[\frac{2k+1}{2}\pi + \frac{2k+1}{2n}\pi\right]}\Bigg|_{n=4} = e^{j(2k+1)\frac{5\pi}{8}}$$

k	s_k
0	$e^{j5\pi/8}$
1	$e^{j15\pi/8} = e^{-j\pi/8}$
-1	$e^{-j5\pi/8}$
2	$e^{j\frac{25\pi}{8}} = e^{j9\pi/8}$
-2	$e^{-j\frac{15\pi}{8}} = e^{j\pi/8}$
3	$e^{j\frac{35\pi}{8}} = e^{j\frac{3\pi}{8}}$
-3	$e^{-j\frac{25\pi}{8}} = e^{-j\frac{\pi}{8}}$
4	$e^{j\frac{45\pi}{8}} = e^{j\frac{5\pi}{8}}$



Choose poles in LHP

$\therefore$ poles: $s_0 = e^{j\frac{5\pi}{8}}$ $\quad s_{-1} = e^{-j\frac{5\pi}{8}}$ $\quad s_{-2} = e^{j\frac{\pi}{8}}$ $\quad s_{-3} = e^{-j\frac{\pi}{8}}$

$$\therefore \quad T(s) = \frac{1}{\left(s - e^{j\frac{5\pi}{8}}\right)\left(s - e^{-j\frac{5\pi}{8}}\right)\left(s - e^{j\frac{\pi}{8}}\right)\left(s - e^{-j\frac{\pi}{8}}\right)}$$

b) Poles: $s_0 = e^{j\frac{5\pi}{8}}$ $\quad s_{-1} = e^{-j\frac{5\pi}{8}}$ $\quad s_{-2} = e^{j\frac{\pi}{8}}$ $\quad s_{-3} = e^{-j\frac{\pi}{8}}$ $\qquad$ Zeros: $\pm\infty$

c) $\omega_c = 2\pi f_c = 4000\pi \text{ rad s}^{-1}$

$\therefore$ Denormalized $T(s) \Rightarrow T_{dn}(s) =$

$$T_{dn}(s) = \frac{15}{\left(\frac{s}{4000\pi} - e^{j\frac{5\pi}{8}}\right)\left(\frac{s}{4000\pi} - e^{-j\frac{5\pi}{8}}\right)\left(\frac{s}{4000\pi} - e^{j\pi/8}\right)\left(\frac{s}{4000\pi} - e^{-j\frac{\pi}{8}}\right)}$$

Poles are scaled: $s = 4000\pi e^{j\frac{5\pi}{8}}, \; 4000\pi e^{-j\frac{5\pi}{8}}, \; 4000\pi e^{j\frac{\pi}{8}}, \; 4000\pi e^{-j\frac{\pi}{8}}$

Zeros: $\pm\infty$

b) Expanding out T(s) from part (a), we obtain:

$$T(s) = \frac{1}{(s^2 + 0.7654s + 1)(s^2 + 1.8478s + 1)}$$

To make a bandpass filter, $s \rightarrow 2p + \frac{1}{2p}$

$$\therefore T_{bp}(p) = \frac{1}{\left[\left(2p + \frac{1}{2p}\right)^2 + 0.7654\left(2p + \frac{1}{2p}\right) + 1\right]\left[\left(2p + \frac{1}{2p}\right)^2 + 1.8478\left(2p + \frac{1}{2p}\right) + 1\right]}$$

$$= \frac{16p^4}{(16p^4 + 6.123p^3 + 12p^2 + 1.531p + 1)(16p^4 + 14.782p^3 + 12p^2 + 3.696p + 1)}$$

Zeros: Four repeated $p = 0$

Poles: $(16p^4 + 6.123p^3 + 12p^2 + 1.531p + 1)(16p^4 + 14.782p^3 + 12p^2 + 3.696p + 1) = 0$

$$\Rightarrow \quad p = \begin{cases} -0.2794 \pm 0.5519i \\ -0.1825 \pm 0.3606i \\ -0.1363 \pm 0.7749i \\ -0.0550 \pm 0.3129i \end{cases}$$

By making the conversion from low pass to band pass, capacitors and inductors in the circuit would need to be modified to reflect the high pass component e.g. a capacitor may need to be replaced with a parallel network of Ls and Cs and inductors will a series network of Ls and Cs. If OTAs are involved we may now need cascaded OTAs to get the band pass component.