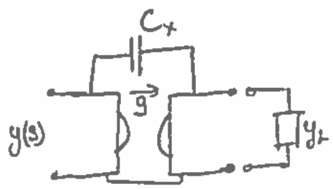
Homework 4

ENEE 610

Problem 1

$$y(s) = \frac{as + b}{s + c}$$

From lecture:



$g^2 = y(k)^2 \qquad C_x = \frac{y(k)}{k}$

→ choose k such that $Ev\{y(s)\} = 2Ev\{y(s)\} = 0$

$$2Ev\{y(s)\} = \frac{as+b}{s+c} + \frac{b-as}{c-s} = \frac{2bc - 2as^2}{c^2 - s^2} = 0$$

$\Rightarrow \quad as^2 = bc$

$\Rightarrow \quad s = k = \sqrt{\frac{bc}{a}} \qquad \therefore \; y(k) = \dfrac{\sqrt{abc} + b}{\sqrt{\frac{bc}{a}} + c}$

→ From Richard's function:

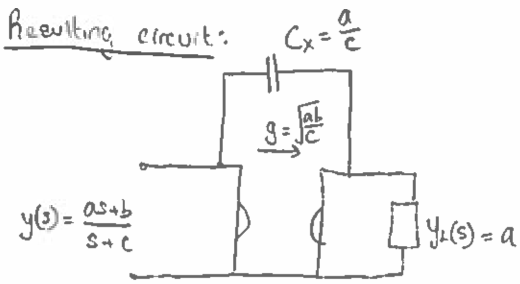
$$y_L(s) = y(k)\,\frac{sy(s) - ky(k)}{sy(k) - ky(s)} \Longrightarrow \text{simplify using } k,\, y(k) \Longrightarrow \boxed{y_L(s) = a}$$

→ Find g, C:

$$g^2 = y(k)^2 = \left(\frac{\sqrt{abc} + b}{\sqrt{\frac{bc}{a}} + c}\right)^2 = \frac{ab}{c} \Rightarrow \boxed{g = \sqrt{\frac{ab}{c}}}$$

$$C_x = \frac{y(k)}{k} = \frac{\sqrt{abc} + b}{\sqrt{\frac{bc}{a}} + c} \times \frac{1}{\sqrt{\frac{bc}{a}}} \Rightarrow \boxed{C_x = \frac{a}{c}}$$

Resulting circuit:



- if $a = 0$, $g = 0$, $y_L = 0$, $C_x = 0$

  → no current at the ports so we get an open circuit

- if $c = 0$, $g = \infty$, $C_x = \infty$

  → the upper gyrator terminals are shorted $\left(Z_{C_x} = \frac{1}{\infty} = 0\right)$ and with $g \to \infty$, the entire device is shorted to bottom terminal

- if $a = -b = c = -1$, $g = 1$, $y_L = -1$, $C_x = 1$

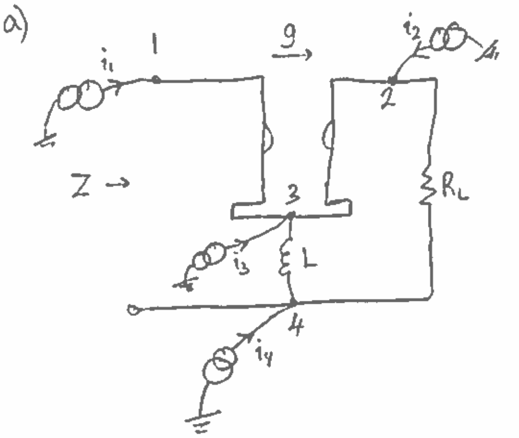
  → the load and device is a negative resistor

- if $a = -b = -c = -1$, $g = \sqrt{-1}$, $y_L = -1$, $C_x = -1$

  → we get both a negative capacitor and resistor with i now introducing a phase shift

Problem 2

a)



Writing current equations:

$$\left.\begin{aligned} i_1 &= g(V_2 - V_3) \\ i_2 &= -g(V_1 - V_3) + G_L(V_2 - V_4) \\ i_3 &= -g(V_2 - V_3) + g(V_1 - V_3) + \frac{1}{sL}(V_3 - V_4) \end{aligned}\right\} \quad i = Y_{indef} V$$

$G_L = 1/R_L$

From these equations write indefinite admittance matrix

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & g & -g & 0 \\ -g & G_L & g & -G_L \\ g & -g & 1/sL & \frac{-1}{sL} \\ 0 & -G_L & \frac{-1}{sL} & G_L + \frac{1}{sL} \end{bmatrix}}_{Y_{indef}} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

last row obtained from sum along columns = 0

b) Grounding node 4:

$$\therefore \quad Y_{indef} = \begin{bmatrix} 0 & g & -g \\ -g & G_L & g \\ g & -g & 1/sL \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

Eliminate node 3:

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \implies i_3 = Y_{21} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + Y_{22} V_3 = 0 \implies V_3 = -Y_{22}^{-1} Y_{21} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

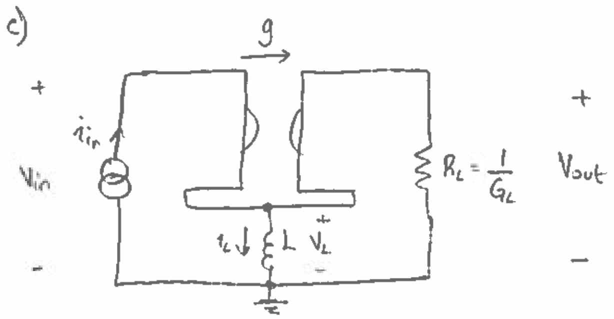
$$\therefore \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = Y_{11} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + Y_{12} V_3 = \underbrace{\left(Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}\right)}_{Y_2} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \implies Y_2 = \begin{bmatrix} 0 & g \\ -g & G_L \end{bmatrix} - \begin{bmatrix} -g \\ g \end{bmatrix} sL \begin{bmatrix} g & -g \end{bmatrix} = \begin{bmatrix} sLg^2 & g - sLg^2 \\ -g - sLg^2 & G_L + sLg^2 \end{bmatrix}$$

Eliminate node 2:

Similarly: $i_1 = Y_{11} V_1 + Y_{12} V_2 = \left(Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}\right) V_1 = \left[sLg^2 - (g - sLg^2) \frac{1}{G_L + sLg^2} (-g - sLg^2) \right] V_1$

$$\implies \boxed{y(s) = \frac{i_1}{V_1} = \frac{g^2(1 + sLG_L)}{G_L + sLg^2}}$$

c)



KCL / KVL Equations

$$\left.\begin{aligned} i_{in} &= g(V_{out} - V_L) \\ i_L &= i_{in} - g(V_{in} - V_L) = i_{in} - \frac{V_{out}}{R_L} \\ V_L &= L\frac{di_L}{dt} \end{aligned}\right\} \quad \begin{aligned} V_L &= V_{out} - \frac{1}{g} i_{in} \\ V_{out} &= (i_{in} - i_L) R_L \end{aligned}$$

$$L\frac{di_L}{dt} = V_{out} - \frac{1}{g} i_{in} = (i_{in} - i_L) R_L - \frac{1}{g} i_{in} = -R_L i_L + \left(R_L - \frac{1}{g}\right) i_{in}$$

$$\left.\begin{aligned} &\Rightarrow \frac{di_L}{dt} = \frac{-R_L}{L} i_L + \frac{1}{L}\left(R_L - \frac{1}{g}\right) i_{in} \Rightarrow \dot{x} = Ax + Bu \\ &V_{out} = -i_L R_L + R_L i_{in} \Rightarrow y = Cx + Du \end{aligned}\right\} \quad \begin{aligned} A &= \frac{-R_L}{L} \qquad B = \frac{1}{L}\left(R_L - \frac{1}{g}\right) \\ C &= -R_L \qquad D = R_L \end{aligned}$$

$$\frac{V_{out}}{i_{in}} = T(s) = C(s - A)^{-1}B + D$$

$$= -R_L \left(\frac{1}{s + R_L/L}\right) \frac{1}{L}\left(R_L - \frac{1}{g}\right) + R_L$$

$$\boxed{T(s) = \frac{R_L\left(\frac{1}{g} + sL\right)}{sL + R_L}}$$