

# Problem #1

## Homework #3

$$Y_{def} = \left[ \begin{array}{ccc|c} g_g & 0 & -g_g & 0 \\ 0 & 0 & g_m & -g_m \\ -g_g & 0 & g_g + sC_g & -sC_g \\ \hline 0 & 0 & -sC_g - g_m & g_s + sC_g + g_m \end{array} \right] = \left[ \begin{array}{c|c} Y_{11} & Y_{12} \\ \hline Y_{21} & Y_{22} \end{array} \right]$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ \hline Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$

Set  $i_4 = 0$ , then:

$$i_4 = Y_{21} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + Y_{22} v_4 \rightarrow v_4 = -Y_{22}^{-1} Y_{21} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = Y_{11} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + Y_{12} v_4 = \underbrace{\left( Y_{11} - Y_{12} Y_{22}^{-1} Y_{21} \right)}_{Y_{3 \times 3}} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$$Y_{3 \times 3} = \begin{bmatrix} g_g & 0 & -g_g \\ 0 & 0 & g_m \\ -g_g & 0 & g_g + sC_g \end{bmatrix} - \frac{1}{g_s + sC_g + g_m} \begin{bmatrix} 0 \\ -g_m \\ -sC_g \end{bmatrix} \begin{bmatrix} 0 & 0 & -sC_g - g_m \end{bmatrix}$$

$$Y_{3 \times 3} = \left[ \begin{array}{cc|c} g_g & 0 & -g_g \\ 0 & 0 & \frac{g_m g_s}{g_s + sC_g + g_m} \\ \hline -g_g & 0 & \frac{g_g(g_s + sC_g + g_m) + sC_g g_s}{g_s + sC_g + g_m} \end{array} \right]$$

Similarly, we find  $Y_{2-port}$ :

$$Y_{2-port} = Y_{11} - Y_{12} Y_{22}^{-1} Y_{21}$$

$$= \begin{bmatrix} g_g & 0 \\ 0 & 0 \end{bmatrix} - \frac{g_s + sC_g + g_m}{g_g(g_s + sC_g + g_m) + sC_g g_s} \begin{bmatrix} -g_g \\ \frac{g_m g_s}{g_s + sC_g + g_m} \end{bmatrix} \begin{bmatrix} -g_g & 0 \end{bmatrix}$$

$$= g_g (1 - \frac{g_s}{g_s + sC_g + g_m})$$

$$Y_{2\text{-port}} = \begin{bmatrix} g_g & 0 \\ 0 & 0 \end{bmatrix} - \frac{g_s + s g_g + g_m}{g_g(g_s + s g_g + g_m) + s g_g g_s} \begin{bmatrix} g_g^2 & 0 \\ \frac{-g_m g_s g_g}{g_s + s g_g + g_m} & 0 \end{bmatrix}$$

$$Y_{2\text{-port}} = \begin{bmatrix} \frac{s g_g g_s g_g}{g_g(g_s + s g_g + g_m) + s g_g g_s} & 0 \\ \frac{g_m g_s g_g}{g_g(g_s + s g_g + g_m) + s g_g g_s} & 0 \end{bmatrix}$$

From notes 9/24/13,  $i_3 = 0, i_4 = 0$ :

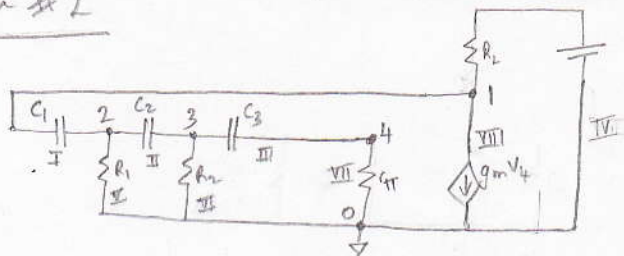
$$Y_{2\text{-port}} = \begin{bmatrix} g_g & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} -g_g & 0 \\ g_m & -g_m \end{bmatrix} \begin{bmatrix} g_g + s g_g & -s g_g \\ -s g_g - g_m & g_g + s g_g + g_m \end{bmatrix}^{-1} \begin{bmatrix} -g_g & 0 \\ 0 & 0 \end{bmatrix}$$

$$Y_{2\text{-port}} = \begin{bmatrix} \frac{s g_g g_s g_g}{g_g(g_s + s g_g + g_m) + s g_g g_s} & 0 \\ \frac{g_m g_s g_g}{g_g(g_s + s g_g + g_m) + s g_g g_s} & 0 \end{bmatrix}$$

• results are the same in both cases. First method may be easier if finding  $Y_{22}^{-1}$  is difficult. However the multiple steps may be another problem.

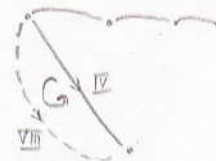
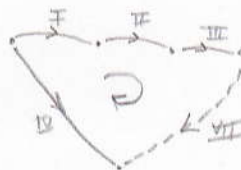
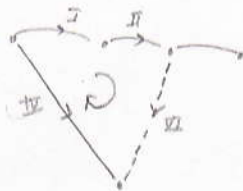
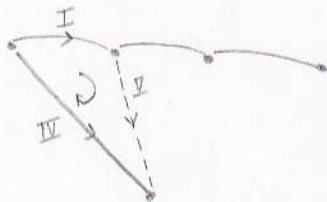
# Problem #2

a)



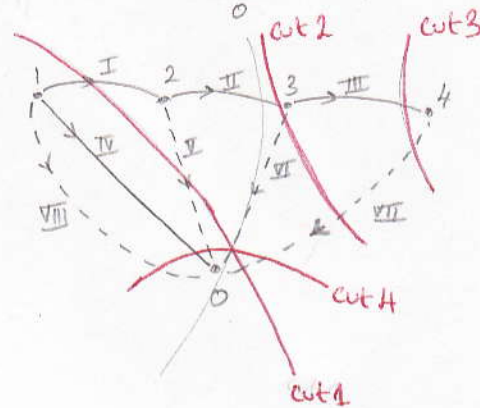
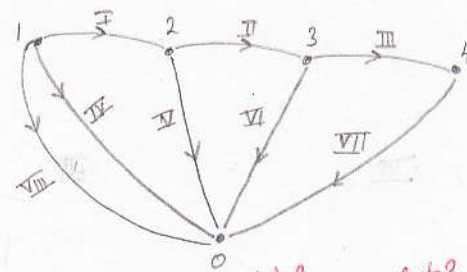
$$C = \left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{array} \right]$$

Cutsat.



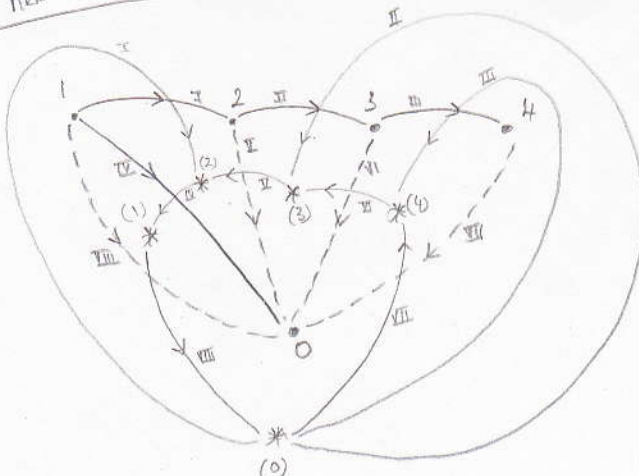
$$T = \left[ \begin{array}{cccc|cccc} 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{array} \right]$$

Tiesat.

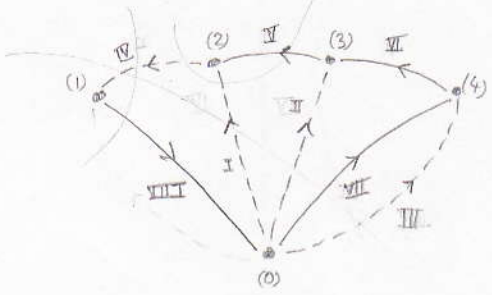


b) Graph does not have crossing branches  $\therefore$  it is planar.

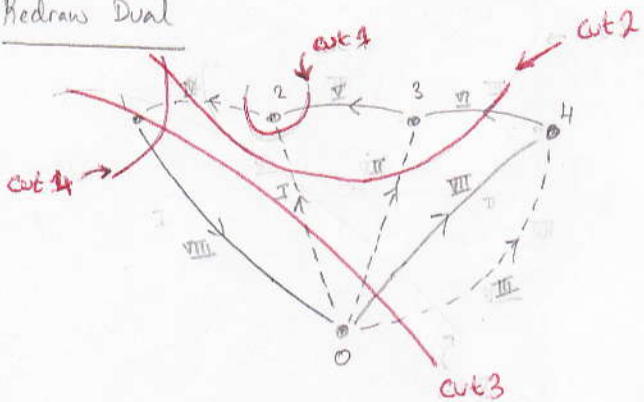
c)



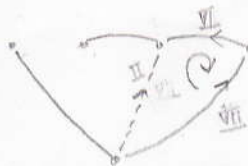
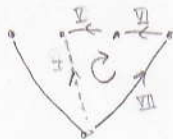
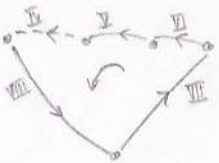
Remove original graph



## Redraw Dual



$$C = \begin{bmatrix} 1 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$



$$\uparrow = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

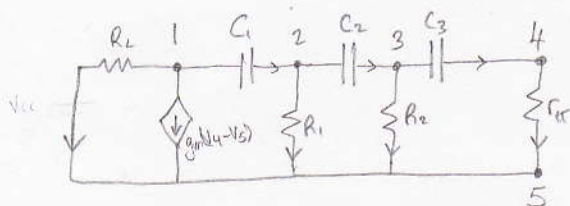
d) The cutset matrix of the original is equal to the transpose of the dual tie-set

" " " " " " " "

cutset " " "

$\therefore$  Deal is original.

# Problem #3



for  $C_1$

$$\begin{aligned} i_1 &= sC_1(V_1 - V_2) \\ i_2 &= sC_1(V_2 - V_1) \end{aligned}$$

$C_2$

$$\begin{aligned} i_2 &= sC_2(V_2 - V_3) \\ i_3 &= sC_2(V_3 - V_2) \end{aligned}$$

$C_3$

$$\begin{aligned} i_3 &= sC_3(V_3 - V_4) \\ i_4 &= sC_3(V_4 - V_3) \end{aligned}$$

$r_{\pi}$

$$\begin{aligned} i_4 &= G_{\pi}(V_4 - V_5) \\ i_5 &= G_{\pi}(V_5 - V_4) \end{aligned}$$

$R_2$

$$\begin{aligned} i_3 &= G_2(V_3 - V_5) \\ i_5 &= G_2(V_5 - V_3) \end{aligned}$$

$R_1$

$$\begin{aligned} i_2 &= G_1(V_2 - V_5) \\ i_5 &= G_1(V_5 - V_2) \end{aligned}$$

• from the current equations on the right, we add the coefficients for each voltage multiplication

$g_m$

$$\begin{aligned} i_1 &= g_m(V_4 - V_5) \\ i_5 &= g_m(V_5 - V_4) \end{aligned}$$

$R_L$

$$\begin{aligned} i_1 &= G_L(V_1 - V_5) \\ i_5 &= G_L(V_5 - V_1) \end{aligned}$$

• check by  $\sum \text{rows} = \sum \text{columns} = 0$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} (sC_1 + G_L) & -sC_1 & 0 & g_m & -g_m - G_L \\ -sC_1 & (sC_1 + sC_2 + G_1) & -sC_2 & 0 & -G_1 \\ 0 & -sC_2 & (sC_2 + sC_3 + G_2) & -sC_3 & -G_2 \\ 0 & 0 & -sC_3 & (sC_3 + G_{\pi}) & -G_{\pi} \\ -G_L & -G_1 & -G_2 & (-g_m - G_{\pi}) & (G_1 + G_2 + G_{\pi} + g_m + G_L) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix}$$

$Y_{\text{indef}}$

• Ground node 5:

$$Y_{4 \times 4} = \begin{bmatrix} sC_1 + G_L & -sC_1 & 0 & g_m \\ -sC_1 & sC_1 + sC_2 + G_1 & -sC_2 & 0 \\ 0 & -sC_2 & sC_2 + sC_3 + G_2 & -sC_3 \\ 0 & 0 & -sC_3 & sC_3 + G_{\pi} \end{bmatrix}$$

• for characteristic equation:

- can find A, B, C, E matrices  $\rightarrow$  state equations
- then  $\det(sI - A)^T$