

EE 303 Midterm solutions F2012 v.1 of 3

$$\#1. \quad a) \quad I_D = \frac{4 \times 10^{-4}}{2} \cdot 1 \cdot (5-1)^2 (1 + 0.01 \times 5) = \frac{2 \times 10^{-4}}{2} \times \left(\frac{W}{L}\right)_p (10-5-1)^2 (1 + 0.01[10-5])$$

$$\Rightarrow \frac{4}{2} = \left(\frac{W}{L}\right)_p \Rightarrow \left(\frac{W}{L}\right)_p = 2$$

b) Need  $v_0 > 5-1=4$  for NMOS in saturation; and for PMOS

$$V_{DD} - v_0 = 10 - v_0 = V_{SDP} \geq V_{SGP} - (-V_{TP}) = V_{DD} - V_{DD} - 1 = 4 \Rightarrow 10 - v_0 \geq 4$$

$$\Rightarrow v_0 \leq 6$$

$\therefore 4 \leq v_0 \leq 6 \Rightarrow M_n \& M_p$  in saturation

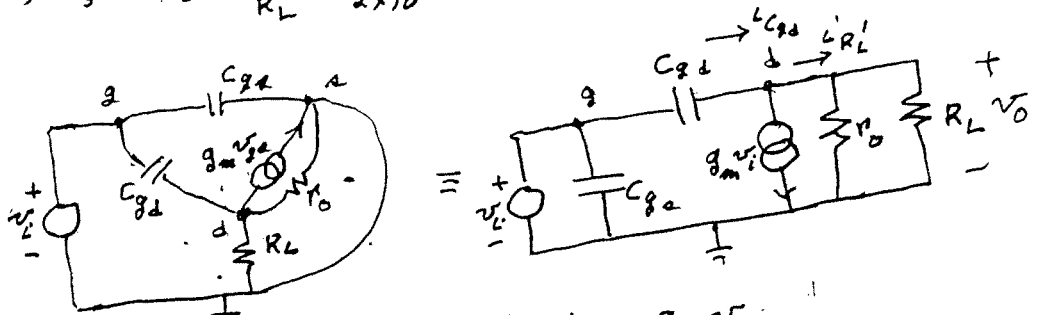
$$c) \quad @ v_0 = 4 \Rightarrow \frac{4}{2} (1 + 0.01 \times 4) = \left(\frac{W}{L}\right)_p (1 + 0.01[10-4]) \Rightarrow \left(\frac{W}{L}\right)_p = \frac{2.08}{1.06} = 1.962$$

$$@ v_0 = 6 \Rightarrow \frac{4}{2} (1 + 0.01 \times 6) = \left(\frac{W}{L}\right)_p (1 + 0.01[10-6]) \Rightarrow \left(\frac{W}{L}\right)_p = \frac{2.12}{1.04} = 2.038$$

$\therefore 1.962 \leq \left(\frac{W}{L}\right)_p \leq 2.038$  gives  $4 \leq v_0 \leq 6 \Rightarrow M_p \& M_n$  in saturation

#2. a) as  $V_0 = 2$ ,  $I_S = -I_D = \frac{V_0}{R_L} = \frac{2}{2 \times 10^3} = 1 \times 10^{-3} = 1 \text{ mA}$

b)



Let  $R_L' = R_L \parallel R_0 = \frac{R_L R_0}{R_L + R_0}$  then  $L_{R_L'} = C_{gd} - g_m v_i$   
 $= \frac{1}{G_L'}$   $G_L' v_o = \frac{1}{R_L'} v_o = \lambda C_{gd} (v_i - v_o) - g_m v_i$   
 $\Rightarrow (G_L' + \lambda C_{gd}) v_o = (\lambda C_{gd} - g_m) v_i$

$\Rightarrow \frac{v_o}{v_i}(s) = \frac{\lambda C_{gd} - g_m}{\lambda C_{gd} + G_L'}$   
 $= \frac{\lambda - g_m / C_{gd}}{\lambda + G_L' / C_{gd}}$

c) pole @  $s_p = -G_L' / C_{gd}$ , zero @  $s_z = g_m / C_{gd}$

Need  $g_m$  &  $g_0$ :  $g_m = \frac{2 \cdot I_S}{(V_{SG} - |V_{TD}|_p)}$ ;  $I_S = 1 \times 10^{-3}$   
 $= \frac{2 \times 10^{-3}}{1} = 2 \text{ mV}$   
 $V_{SG} = 10 - 8 = 2$

$g_0 = \frac{\lambda I_S}{1 + \lambda V_{SD}} = \frac{10^{-5}}{1 + 0.01 \times 8} = \frac{10^{-5}}{1.08} = \approx 2 I_S = 10^{-5}$

as  $G_L = \frac{1}{2} \times 10^{-3}$ ,  $G_L' = G_L + g_0 \approx 0.5 \times 10^{-3} + 10^{-5} = 51 \times 10^{-5}$

and as  $C_{gd} = 5 \text{ pF} = 5 \times 10^{-12}$

$s_p = -51 \times 10^{-5} / 5 \times 10^{-12} = -10.2 \times 10^7 = -102 \times 10^6$

$s_z = 2 \times 10^{-3} / 5 \times 10^{-12} = \frac{2}{5} \times 10^9 = 4 \times 10^8 = 400 \times 10^6$

#3. a) @ DC no current in  $C_{pb} \Rightarrow$

$$\text{MOS: } I_D = 2 \text{ mA} \Rightarrow V_O = V_{DD} - R_L I_D = 10 - 2 \times 10^{-3} \times 2 \times 10^3 = 6 \text{ V}$$

$$\text{BJT: need } \alpha = \frac{\beta}{1+\beta} = \frac{99}{100} = 0.99 \Rightarrow I_C = \alpha \times 2 \times 10^{-3} = 1.98 \times 10^{-3}$$

$$\Rightarrow V_O = 10 - 2 \times 10^{-3} \times 2 \times \frac{99}{100} \times 10^3 = 10 - 3.76 = 6.24$$

$\therefore V_{O_{\text{MOS}}} = 6 \text{ V} < 6.24 = V_{O_{\text{BJT}}}$  due to  $I_C < I_D = I_{\text{bias}}$

b) MOS: need  $V_{GS}$ :  $I_D = 2 \times 10^{-3} = \frac{4 \times 10^{-4}}{2} \cdot 1 (V_{GS} - 1)^2 (1 + 0.01 [6 - 3])$   
 $\Rightarrow \frac{10}{1.03} = (V_{GS} - 1)^2 \Rightarrow V_{GS} = 1 + \sqrt{10/1.03} = 1 + 3.116$   $V_{DS} = V_O - V_{C_{pb}}$

$$\Rightarrow V_G = V_{C_{pb}} + V_{GS} = 3 + 1 + \sqrt{10/1.03} = 7.116$$

$$= \frac{R_b}{R_a + R_b} V_{DD} = \frac{1}{1 + \frac{100 \times 10^3}{R_b}} \times 10 \Rightarrow 1 + \frac{100 \times 10^3}{R_b} = \frac{10}{7.116} = 1.405$$

$$\Rightarrow \frac{100 \times 10^3}{R_b} = 0.405 \Rightarrow R_b = \frac{100}{0.405} \times 10^3 = 247 \text{ K}\Omega$$

BJT:  $V_B = V_{BE} + V_{C_{pb}} = 3.7 = R_b I_{R_b} \Rightarrow R_b = V_B / I_{R_b} \Rightarrow \text{need } I_{R_b}$

$$I_{R_b} = I_{R_a} - I_B = I_{R_a} - \frac{I_C}{\beta} = I_{R_a} - \frac{\alpha}{\beta} I_{\text{bias}} = I_{R_a} - \frac{1}{\beta+1} I_C$$

$$= I_{R_a} - 2 \times 10^{-5}$$

and

$$V_{CC} = I_{R_a} R_a + V_B \Rightarrow I_{R_a} = \frac{V_{CC} - V_B}{R_a} = \frac{10 - 3.7}{R_a} = \frac{6.3}{R_a} = \frac{6.3}{R_a \times 10^5}$$

$$\Rightarrow I_{R_b} = (6.3 - 2) \times 10^{-5} = 4.3 \times 10^{-5}$$

$$\Rightarrow R_b = \frac{3.7}{4.3} \times 10^5 = 0.837 \times 10^5 = 83.7 \text{ K}\Omega$$

c) MOS:  $g_m = \frac{2 I_D}{(V_{GS} - V_{TO})} = \frac{4 \times 10^{-3}}{\sqrt{10/1.03}} = \frac{4}{3.116} \times 10^{-3} = 1.28 \times 10^{-3}$

$$g_o = \frac{\lambda I_D}{1 + \lambda V_{DS}} = \frac{10^{-2} \times 2 \times 10^{-3}}{1.03} = \frac{2}{1.03} \times 10^{-5} = 1.94 \times 10^{-5} \approx 2 I_D = 2 \times 10^{-5}$$

BJT:  $g_m = \frac{I_C}{V_T} = \frac{\alpha \times 2 \times 10^{-3}}{26 \times 10^{-3}} = \frac{\alpha}{13} = 0.0762$

$$g_o = \frac{I_E}{V_A} = \frac{\alpha \times 2 \times 10^{-3}}{100} = 1.98 \times 10^{-5}$$

$$g_{m_{\text{MOS}}} = 1.2 \times 10^{-3} < 76.2 \times 10^{-3} = g_{m_{\text{BJT}}} \Rightarrow \text{BJT is stronger}$$

$$g_{o_{\text{BJT}}} = 1.98 \times 10^{-5} < 1.94 \times 10^{-5} = g_{o_{\text{MOS}}} \Rightarrow \text{MOS has less loss}$$