

EEEL 610 homework set 5 solutions

11/20/06
K, I

#1. $y(s) = \frac{s^2 + as + 1}{s^2 + bs + 1}$, $a \geq 0, b \geq 0$

Case 1: $a = b \Rightarrow y(s) = 1$ which is PR

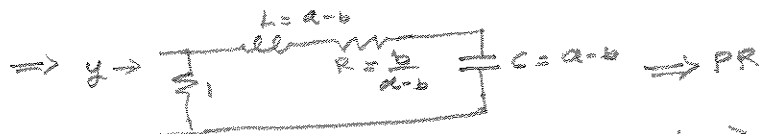
Case 2: $a \neq b$

Case 2a): $b = 0 \Rightarrow y(s) = 1 + \frac{as}{s^2 + 1} \Rightarrow$ PR as $y \rightarrow \frac{1}{s} \parallel C = a > 0$ as $a > 0$

Case 2b): $a = 0 \Rightarrow \frac{1}{y(s)} = 1 + \frac{bs}{s^2 + 1} \Rightarrow$ PR as $y \rightarrow \frac{1}{s} \parallel C = \frac{1}{b}$ as $b > 0$

Case 2c): $a > 0, b > 0, a > b$

$$y(s) = 1 + \frac{(a-b)s}{s^2 + bs + 1} = 1 + \frac{1}{\frac{s}{a-b} + \frac{b}{a-b} + \frac{1}{(a-b)s}}$$



Case 2d: $b > 0, b > a \Rightarrow$ dual of Case 2c) $\Rightarrow$ PR as



$\therefore y(s)$ is PR for all $a \geq 0, b \geq 0$, since it is the driving-point admittance of a passive circuit (= connection of passive components as energy added) in all of these cases
[also true for any real $a = b < 0$! $\Rightarrow y(s)$ is PR if & only if a & b are real and either $a = b$ or $a > 0$ and $b \geq 0$].

#2.

$$y(s) = \frac{s^2 + s + 1}{s^2 + s + 3} = \frac{N(s)}{D(s)}, \quad N(s) = s^2 + s + 1 = (s - \beta_1)(s - \beta_2)$$

$$D(s) = s^2 + s + 3 = (s - \gamma_1)(s - \gamma_2)$$

$$\beta_{1,2} = -\frac{1}{2} \pm \frac{1}{2}\sqrt{1-4} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

$$\gamma_{1,2} = -\frac{1}{2} \pm \frac{1}{2}\sqrt{1-12} = -\frac{1}{2} \pm j\frac{\sqrt{11}}{2}$$

$$\text{RL } y(s) = \left(\frac{s^2 + s + 1}{s^2 + s + 3} \right) + \left(\frac{s^2 - s + 1}{s^2 - s + 3} \right) = \frac{[s^2 + s + 1][s^2 - s + 3] + [s^2 - s + 1][s^2 + s + 3]}{[s^2 + s + 3][s^2 - s + 3]}$$

$$= \frac{[s^4 + 3s^2 - s^2 + s^2 + 3] + [s^4 + s^2 - s^2 + 3s^2 - 3]}{[s^4 + 3s^2 - s^2 + 3s^2 + 9]}$$

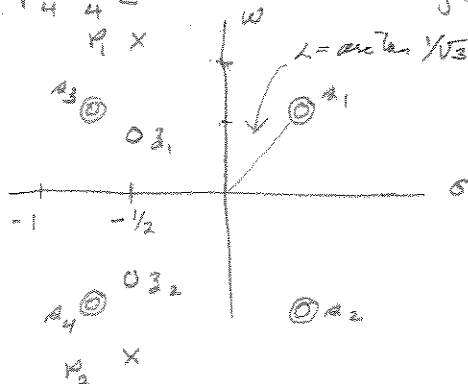
$$= 2[s^4 + 3s^2 + 3] / [s^4 + 5s^2 + 9]$$

$$N_{\text{RL}} = s^4 + 3s^2 + 3 = (s - \alpha_1)(s - \alpha_2)(s - \alpha_3)(s - \alpha_4)$$

Let $x = s^2 \Rightarrow N_0(x) = x^2 + 3x + 3 \Rightarrow x_{1,2} = -\frac{3}{2} \pm \frac{1}{2}\sqrt{9-12} = -\frac{3}{2} \pm j\frac{\sqrt{3}}{2}$

$\therefore x_{1,2} = \sqrt{\frac{9}{4} + \frac{3}{4}} e^{j[\pi \pm \arctan(\sqrt{3})]}$

$\therefore \alpha_{1,2,3,4} = \pm \sqrt{3} e^{j[\pi \pm \arctan(\sqrt{3})]/2}$



= poles x, & zeros o of y
Ev. y zeros, $\odot$ [note: 2 in LHP, 2 in RHP
as have $\alpha_1, \alpha_1^*, -\alpha_1, -\alpha_1^*$]

det 5 solution

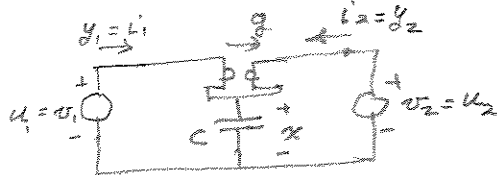
#3: a) $Y(s) = Z^{-1}(s)$; $Z(s) = sL \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -1/g \\ 1/g & 0 \end{bmatrix} = \begin{bmatrix} sL & sL - 1/g \\ sL + 1/g & sL \end{bmatrix}$

$\det Z = (sL)(sL) - (sL - 1/g)(sL + 1/g) = 1/g^2$

$\therefore Y(s) = \begin{bmatrix} sLg^2 & -sLg^2 + g \\ -sLg^2 - g & sLg^2 \end{bmatrix} = sLg^2 \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + g \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

as $sLg^2 v_1 \rightarrow Lg^2 dv_1/dt$ this requires differentiation of the input which can not come from $x = Ax + Bu$, $y = Cx + Du$ ($\equiv Y(s)$) has poles at ∞ which can not be in $T(s) = D + C[1 - sA]^{-1}B$.

b) We replace sL by $\frac{1}{sC}$ in the above: $Y(s) = \frac{g^2}{sC} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} + g \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$



Here

$C\dot{x} = i_1 + i_2 = g[v_2 - x] + (-g)[v_1 - x]$
 $= g(-x + x) + gv_2 - gv_1$
 $= 0 + [-g \quad g] \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

$y = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} gv_2 - x \\ -g(v_1 - x) \end{bmatrix} = \begin{bmatrix} -g \\ g \end{bmatrix} x + \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} u$

$\therefore \dot{x} = 0 + \frac{g}{C} [-1 \quad 1] u$
 $y = g \begin{bmatrix} -1 \\ 1 \end{bmatrix} x + g \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} u$

$T(s) = D + C[s1 - A]^{-1}B = g \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + g \begin{bmatrix} -1 \\ 1 \end{bmatrix} \frac{1}{s} \cdot \frac{g}{C} [-1 \quad 1]$
 $= g \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} + \frac{g^2}{sC} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ which checks $Y(s)$

c) $y_m = y_{11} - y_{12} \frac{1}{y_{22} + G} y_{21} = \frac{g^2}{sC} - (+g - \frac{g^2}{sC}) \frac{1}{\frac{g^2}{sC} + G} (-g - \frac{g^2}{sC})$, $G = 1/R$
 $= \frac{g^2}{sC} + \frac{g^2 - g^4/(sC)^2}{(g^2 + GsC)/sC} = \frac{g^2}{sC} + \frac{g^2}{sC} \frac{[(sC)^2 - g^2]}{g^2 + GsC}$
 $= \frac{g^2}{sC} \left[\frac{GsC + (sC)^2}{g^2 + GsC} \right] = \frac{G + sC}{1 + \frac{G}{g^2} sC}$ (which is degree 1 as expected)

$\therefore z(s) = \frac{1}{y_m} = \frac{1 + sC(G/g^2)}{G + sC}$

2Ew $z(s) = z(s) + z(-s) = \frac{1 + sC(G/g^2)}{G + sC} + \frac{1 - sC(G/g^2)}{G - sC}$
 $= \frac{[1 + sC(G/g^2)][G - sC] + [1 - sC(G/g^2)][G + sC]}{G^2 - (sC)^2}$

$= 2[G - (sC)^2(G/g^2)] / [G^2 - (sC)^2] = 2G[1 - (sC/g)^2] / G[1 - (sC/g)^2]$

$\Rightarrow$ 2w $z(s)$ zeros: $s_{1,2} = \pm \frac{1}{C/g} = \pm g/C$ irrespective of load $G = 1/R$