

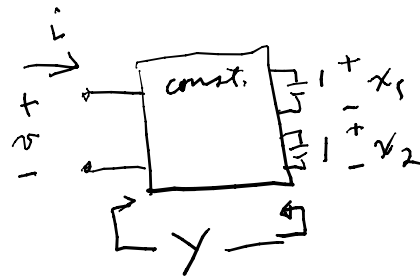
$$T(s) = Y(s) = \frac{s^2 + as + b}{s^2 + cs + d} = \frac{i(s)}{v(s)}$$

11/01/06
ΣΣ 610

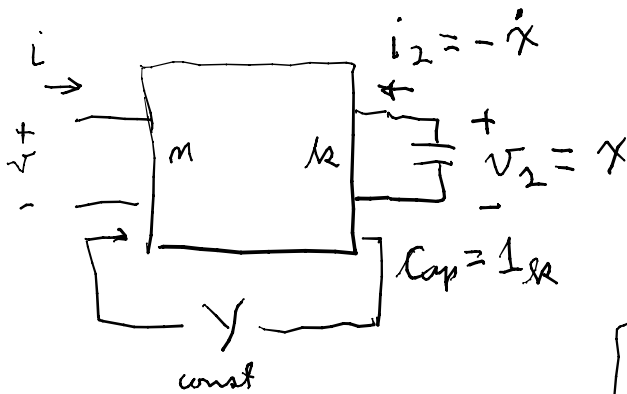
set up state variable eqs.

$$\dot{x} = Ax + Bv(t)$$

$$i(t) = Cx + Dv(t)$$



Ex: here Y is 3×3



$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ \text{coupling} & Y_{21} & Y_{22} \end{bmatrix}$$

$$\begin{bmatrix} i \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

can't be eqs.

$$(-s \mathbf{1}_2 - Y_{22}) v_2 = Y_{21} v_1$$

$$i = (Y_{11} - Y_{12} [s \mathbf{1}_2 + Y_{22}]^{-1} Y_{21}) v_1 = Y(s) v_1$$

$n \times n$

$$i = (D + C [s \mathbf{1}_n - A]^{-1} B) v$$

$$Y(s) = D + C [s \mathbf{1}_n - A]^{-1} B = Y_{11} - Y_{12} [s \mathbf{1}_2 + Y_{22}]^{-1} Y_{21}$$

identify $Y_{11} = D$, $-Y_{12} = C$, $Y_{21} = B$, $Y_{22} = -A$

$$\therefore Y_{\text{coupling}} = \begin{bmatrix} D & -C \\ B & -A \end{bmatrix} \quad (\text{a constant matrix})$$

if $Y(s)$ is PR then desire Y_{coupling} to go with a passive circuit:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

let $x = X\hat{x}$, X^{-1} exists
 $X = \text{const.}$

$$\dot{x} = X\dot{\hat{x}} = AX\hat{x} + Bu \Rightarrow \dot{\hat{x}} = X^{-1}AX\hat{x} + X^{-1}Bu$$

$$y = CX\hat{x} + Du$$

$$\dot{\hat{x}} = \hat{A}\hat{x} + \hat{B}u$$

$$y = \hat{C}\hat{x} + Du$$

$$Y_{\hat{x}} = \begin{bmatrix} D & -C \\ B & -A \end{bmatrix} \Rightarrow \hat{Y}_{\hat{x}} = \begin{bmatrix} D & -\hat{C} \\ \hat{B} & -\hat{A} \end{bmatrix} = \begin{bmatrix} D & -CX \\ X^{-1}B & -X^{-1}AX \end{bmatrix}$$

$$= \begin{bmatrix} I_n & 0 \\ 0 & X^{-1} \end{bmatrix} \begin{bmatrix} D & -C \\ B & -A \end{bmatrix} \begin{bmatrix} I_n & 0 \\ 0 & X \end{bmatrix}$$

$\therefore$ by a choice of X (a $k \times k$ matrix) (use $P^{1/2}$)

can get desired properties of the coupling matrix

If $Y(s)$ PR can find an X to make $Y_{\hat{x}}$ passive.
 (requires solving a Riccati equation).

If $u=0$, $\dot{x} = Ax$, if $x(0) \neq 0$ then a solution

is $x(t) = e^{+At} x(0)$ can find by $L[x] = [sI_k - A]^{-1} x(0)$

denominator of $[sI_k - A]^{-1}$ is $\det[sI_k - A]$ with all zeros in the left half plane for A from a PR matrix with no $j\omega$ axis poles

There exists a positive definite matrix P such that
 given a positive definite matrix M then

$$PA + A^T P = -M \approx -PB^T B P$$

[gives Riccati eq. for P
 given A & B ; if A is
 stable Persids, $P > 0$]

Ex: $y(s) = \frac{s^2 + as + b}{s^2 + cs + d} = \frac{I(s)}{V(s)} = \frac{i}{v} = \frac{y}{u}$

$$(s^2 + as + b)v = (s^2 + cs + d)i$$

$$s(a(i-v) + (ci - av)) + (di - bv) = 0$$

x_1
 x_2

$$-ax_2 = di - bv \quad ; \quad x_2 = ax_1 + ci - av$$

$$x_1 = i - v \Rightarrow y = x_1 + u$$

$$-ax_2 = d(x_1 + u) - bu$$

$$ax_1 = x_2 - c[x_1 + u] + au$$

$$A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -c & 1 \\ -d & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -c+1 \\ -d+b \end{bmatrix} u$$

$$i = y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \end{bmatrix} u = v$$

$$Y_c = \begin{bmatrix} 1 & [-1 & 0] \\ [-c+1] & [c & -1] \\ [-d+b] & [d & 0] \end{bmatrix}$$

$$i_c = Y_c v_c$$

$$i_c^T v_c \geq 0$$

$$i_c^T v_c + v_c^T i_c = v_c^T (Y_c^T) v_c + v_c^T Y_c v_c$$

$$= v_c^T [Y_c^T + Y_c] v_c \geq 0 \text{ if passive.}$$

$$2Y_{sym} = Y_c^T + Y_c \text{ is positive semidefinite}$$

for any v_c

$$2Y_{c,sym} = \begin{bmatrix} 1 & -1 & 0 \\ -1+c & c & -1 \\ -d+b & d & 0 \end{bmatrix} + \begin{bmatrix} 1 & -c+a & -d+b \\ -1 & c & d \\ 0 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1-c+a & -d+b \\ -1-c+a & 2c & d-1 \\ -d+b & d-1 & 0 \end{bmatrix}$$

desire this to be positive semidefinite

test is that a set of nested principle determinants is $\geq$

$$\Delta_{22,33} = 2 > 0, \Delta_{33} = \det \begin{bmatrix} 2 & -1-c+a \\ -1-c+a & 2c \end{bmatrix} = 4c - (1+c-a)^2$$

$$\geq 0 \text{ if } 4c > (1+c-a)^2; \text{ need not be the case}$$

$$\Delta = \det \begin{bmatrix} 2 & -1-c+a & -d+b \\ -1-c+a & 2c & d-1 \\ -d+b & d-1 & 0 \end{bmatrix}$$

$$c=1, a=5$$

We know $y(s) = \frac{s^2 + s + 1}{s^2 + s + 2}$ is PR; $a=b=c=1, d=2$

$$Y_c = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 2 & 0 \end{bmatrix}; 2Y_{c,sym} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix}; \Delta_{22,33} = 2, \Delta_{33} = 3$$

$$\Delta = (-1)(-1)^{3+1} \begin{vmatrix} -1 & 2 \\ -1 & 1 \end{vmatrix} + (1)(-1)^{3+2} \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} = -(-1+2) + 1(-1) \cdot 1 = -1 - 1 = -2 < 0$$

here $Y_{\lambda \text{ sym}}$ is ~~not~~ positive semi-definite, is ~~actually~~ ^{not} passive

$$Y_{\lambda} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 2 & 0 \end{bmatrix} = \underbrace{\frac{1}{2} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix}}_{\frac{Y_c + Y_c^T}{2}} + \underbrace{\frac{1}{2} \begin{bmatrix} 0 & -1 & +1 \\ +1 & 0 & -3 \\ -1 & 3 & 0 \end{bmatrix}}_{\text{all gyrators}}$$

form

$$\frac{1}{2} T^T \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix} T = \frac{1}{2} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 1 & 0 \\ -1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & -1 \\ 1 & 1 & 0 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & +1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & +1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & +1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & & \\ & 2 & \\ & & -1/2 \end{bmatrix} \leftarrow \begin{matrix} \text{use a} \\ \text{neg. resistor} \end{matrix}$$

$$\frac{1}{2} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix} = \frac{1}{2} T^{-T} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1/2 \end{bmatrix} T^{-1}$$

$$T^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -1/2 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \Rightarrow \text{made by transformers}$$

$$v_1 = T^{-1} v_2$$

But can find an X to make $\hat{\gamma}_c$ passive (ie. PR)