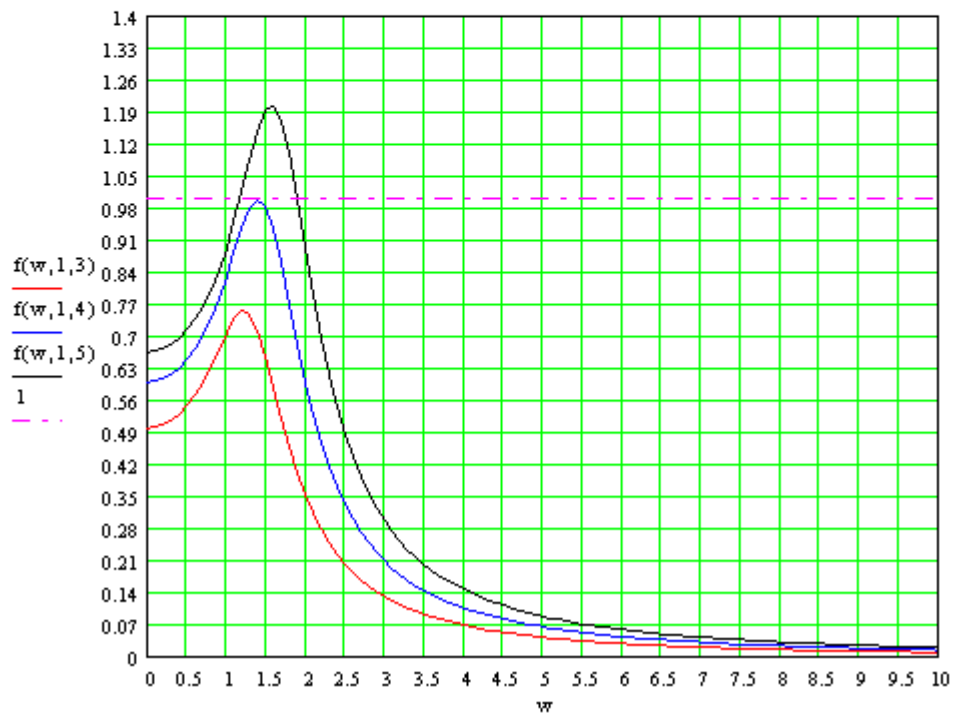


10/25/06
ΣΕ610

$$y(s, a, b) := \frac{(s^2 + s + a)}{(s^2 + s + b)} \quad S(s, a, b) := \frac{(1 - y(s, a, b))}{(1 + y(s, a, b))}$$

$$f(w, a, b) := |S(j \cdot w, a, b)|$$

$$w := 0, 0.1 \dots 10$$



Synthesis of any PR function:

(Brune, p. 352 uses transformers (uses min. # of L, C's)
Bott-Duffin, p. 361, uses no transformers but very many L, C, R's

Darlington, min # of L, C's, uses transformers, uses only one resistor (as the load)

alternate: Find the even part zeros, remove

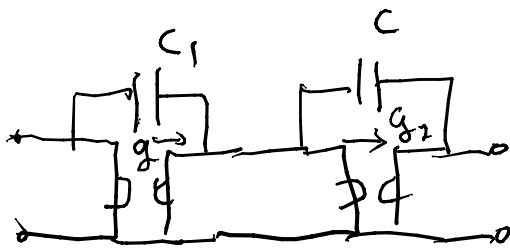
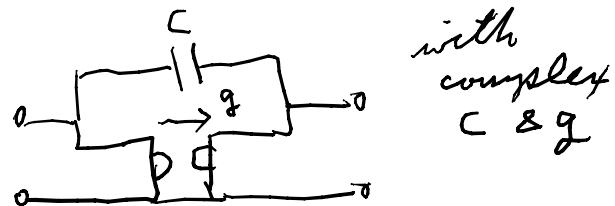
one at $s = k = k_r + j k_i$ using Richards' function

$y(s)$, $y_L(s)$ which has complex coefficients

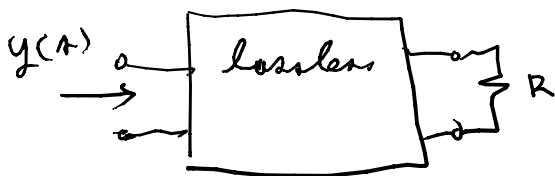
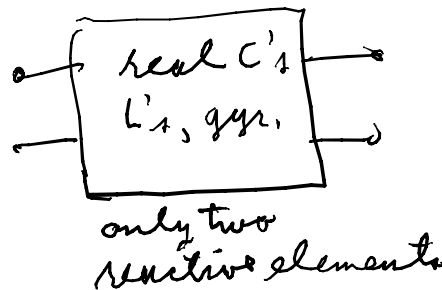
repeat on $y_L(s)$ by removing a zero of the even part at $s = k^* = k_r - j k_i$; then $y_{LL}(s)$

if $y(s)$ is PR then $y_{LL}(s)$ is PR, $S[y_{LL}] = S[y] - 2$

if $k_i \neq 0$ this uses



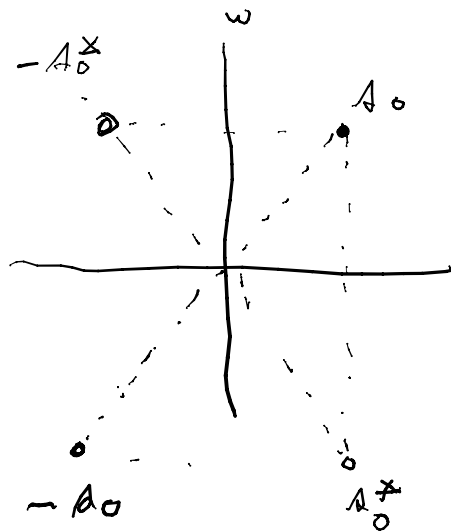
$\Rightarrow$



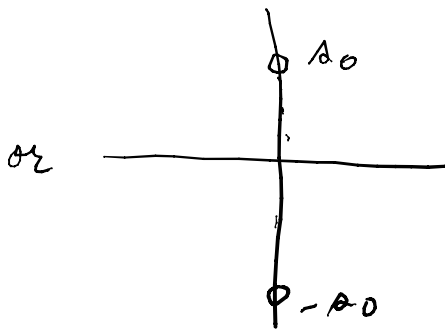
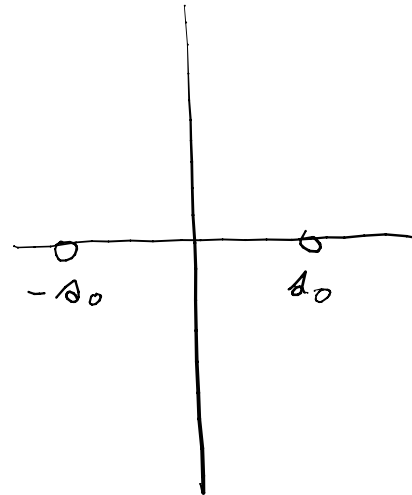
if $y(s)$ is PR & of degree n then uses only n reactive elements.

Even part zeros: $2 \operatorname{Ev} y(s) = y(s) + y(-s)$

if s_0 is a zero of $\operatorname{Ev} y(s) = y(s_0) + y(-s_0) = 0$
shows $-s_0$ is also a zero of $\operatorname{Ev} y(s)$



σ ω



$$E_f: y(s) = \frac{s^2 + s + a}{s^2 + s + b}$$

$$2E_f y(s) = \frac{s^2 + s + a}{s^2 + s + b} + \frac{s^2 - s + a}{s^2 - s + b} = \frac{(s^2 + s + a)(s^2 - s + b) + (s^2 - s + a)(s^2 + s + b)}{(s^2 + s + b)(s^2 - s + b)}$$

Zeros of $E_f y(s)$ are zeros

$$N(s) = 2[s^4 + bs^2 - s^2 + as^2 + ab]$$

$$\frac{N(s)}{2} = (s^2)^2 + ((b+a)-1)(s^2) + ab; \quad x = s^2$$

$$= x^2 + cx + d; \quad x = -\frac{((b+a)-1)}{2} \pm \frac{1}{2} \sqrt{((b+a)-1)^2 - 4ab}$$

$$s \mid \text{zero of} = \pm \sqrt{\frac{-(b+a)-1}{2} \pm \frac{1}{2} \sqrt{((b+a)-1)^2 - 4ab}}$$

even part gives 4 roots

$$\begin{aligned} 2 \operatorname{Re} y(s) &= y(s) + y^*(s) = y(s) + y(s^*) \\ 2 \operatorname{Ev} y(s) &= y(s) + y(-s) \end{aligned} \quad \left. \vphantom{\begin{aligned} 2 \operatorname{Re} y(s) &= y(s) + y^*(s) \\ 2 \operatorname{Ev} y(s) &= y(s) + y(-s) \end{aligned}} \right\} \begin{array}{l} \text{equal if} \\ s = j\omega \end{array}$$

State Variables:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

x = state vector, $\dot{} = d/dt$

u = input

y = output

if linear & time invariant

A, B, C, D are constant

does not allow differentiators for y vs u ; $y = \dot{u}$

Semi-state variables

$$E \dot{x} = Ax + Bu$$

$$y = Cx$$

x = semi-state
(= state if $E = I$ = identity)