

Richards' function, p. 361

10/16/06
EE610

$$R(s) = \frac{k z(s) - A z(k)}{k z(k) - A z(s)}$$

if $z(s)$ is PR & $k > 0$

then $R(s)$ is PR

$$\frac{1}{R(s)}, z(k) \text{ is } > 0$$

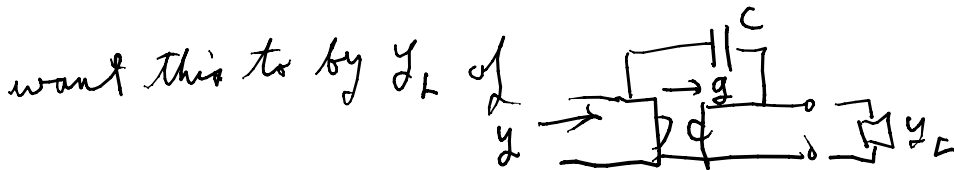
looks like a pole at $s = k$



but a zero is at $s = k$
at $s = k$, $k z(k) - k z(k) = \text{numerator}$
in den $k z(k) - k z(k)$

$$\text{if } z(s) = 1/y(s) \text{ then } R(s) = \frac{k/y(s) - A/y(k)}{k/y(k) - A/y(s)} = \frac{k y(k) - A y(s)}{k y(s) - A y(k)}$$

$$\text{let } y_R(s) = y(k) R(s) = y(k) \left[\frac{k y(k) - A y(s)}{k y(s) - A y(k)} \right]$$



$$Y_{g-C} = \begin{bmatrix} sC & -sC + g \\ -sC - g & sC \end{bmatrix} \text{ load in } Y_L, \Delta_Y = g^2$$

$$y_i = y_{in} = y_{11} - y_{12} (y_{22} + y_L)^{-1} y_{21} = \frac{\Delta_Y + y_{11} y_L}{y_{22} + y_L}$$

$$y_i y_{22} + y_i y_L = \Delta_Y + y_{11} y_L \Rightarrow (y_i - y_{11}) y_L = \Delta_Y - y_{22} y_i$$

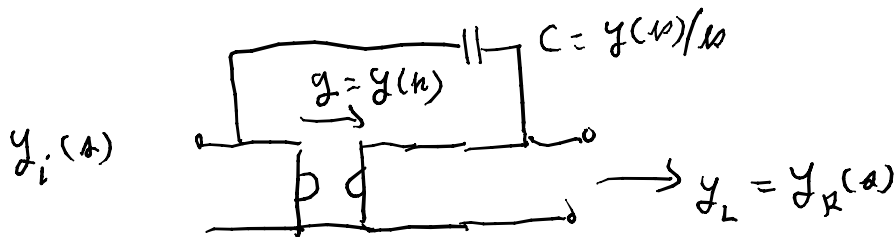
$$y_L(s) = \frac{\Delta_Y - y_{22} y_i}{-y_{11} + y_i} = \frac{g^2 - sC y_i(s)}{-sC + y_i(s)} \Leftrightarrow y_R(s) = y(k) \left[\frac{k y(k) - A y(s)}{k y(s) - A y(k)} \right]$$

$$y_L(s) = \frac{C}{C} \left[\frac{(g^2/C) - s y_i(s)}{\frac{y_i(s)}{C} - s} \right] = y_R(s) = \frac{y(s)}{y(s)} \left[\frac{k y(k) - s y(s)}{\frac{k}{y(k)} y(s) - s} \right]$$

Set $y(s) = y_i(s)$, $\frac{1}{C} = \frac{k}{y(k)}$, $\frac{g^2}{C} = k y(k)$

$$C = \frac{y(k)}{k}, \quad g^2 = C(k y(k)) = \frac{y(k)}{k} \cdot k y(k) = y(k)^2$$

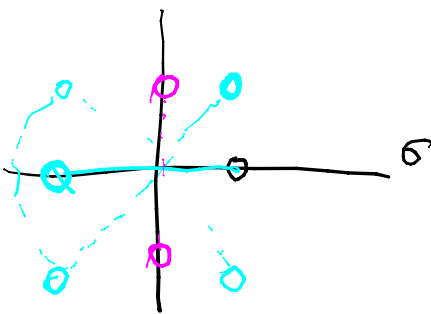
$$g = + y(k) \text{ or } - y(k)$$



here
 $\delta[y_R] \leq \delta[y_i]$

at an even part zero of $y(s)$: $2\text{Ev } y(s) = y(s) + y(-s)$

then $y(s) = -y(s)$ at an even part zero



But for a reactance function $y(s)$
 $y(s)$ is odd, $y(s) = -y(-s)$ for
 all s ; $\therefore$ can choose any k and
 $y(k) = -y(-k)$ if $y(s)$ is lossless PR

$$\text{Look at } s = -k \text{ in } y_R(s) = y(k) \left[\frac{k y(k) - s y(s)}{k y(s) - s y(k)} \right] \Big|_{s=-k}$$

$$= y(k) \left[\frac{k y(k) + k y(-k)}{k y(-k) + k y(k)} \right]_{s=-k}$$

if $y(-k) = -y(k)$ the numerator is zero & also the denominator: $\therefore (s-k)(s+k) = s^2 - k^2$ cancels in numerator & denominator $\Rightarrow \delta[y_k] \leq \delta[y_i] - 1$

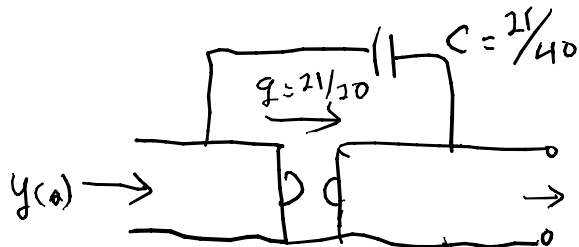
Ex: $y(s) = \frac{3s(s^2+3)}{(s^2+1)(s^2+4)}$, $\delta[y] = 4$; $\delta[\cdot] = \text{degree (highest power of } s \text{ in } y)$

$y(s) = -y(-s)$

choose a $k > 0$, say $k = 2$;

$$y(k) = \frac{3 \times 2 \times (4+3)}{(4+1)(4+4)} = \frac{6 \times 7}{5 \times 8} = \frac{21}{20}$$

$$C = \frac{y(k)}{k} = \frac{21}{20 \times 2} = \frac{21}{40}$$



$$y_R = \frac{21}{20} \left[\frac{\frac{21}{10} - s \left(\frac{3s(s^2+3)}{(s^2+1)(s^2+4)} \right)}{\frac{2 \times 3s(s^2+3)}{(s^2+1)(s^2+4)} - \frac{21}{20} s} \right] = \frac{21}{20} \left[\frac{\frac{21}{10}(s^4+5s^2+4) - 3s^4 - 9s^2}{s \{ 6s^2 + 18 - (\frac{21}{20})[s^4+5s^2+4] \}} \right]$$

$$= \frac{21}{20} \left[\frac{-\frac{9}{10}s^4 + \frac{3}{2}s^2 + \frac{42}{5}}{s \{ -\frac{21}{20}s^4 + \frac{3}{4}s^2 + \frac{69}{5} \}} \right]$$

$$6 - \frac{21}{4}$$

$$\frac{18}{5} - \frac{21}{60} = \frac{69}{60}$$

we know $s-2$ & $s+2$ cancel $\Rightarrow s^2-4$ divides num. & den.

$$s^2 - k^2 = (s-k)(s+k)$$

$$\begin{array}{r}
 s^2 - 4 \quad \left| \begin{array}{r} -\frac{9}{10}s^2 - \frac{21}{10} \\ -\frac{9}{10}s^4 + \frac{3}{2}s^2 + \frac{42}{5} \\ +\frac{36}{10}s^2 \\ \hline -\frac{21}{10}s^2 + \frac{84}{10} = \frac{42}{5} \end{array} \right.
 \end{array}$$

$$\begin{array}{r}
 36 \\
 -15 \\
 \hline
 21
 \end{array}$$

$$\begin{array}{r}
 s^2 - 4 \quad \left| \begin{array}{r} -\frac{21}{20}s^2 - \frac{306}{40} \frac{69}{20} \\ -\frac{21}{20}s^4 + \frac{3}{4}s^2 - \frac{69}{5} \\ -\frac{21}{10}s^4 + \frac{84}{20}s^2 \\ \hline -\frac{306}{40}s^2 + \frac{69}{5} \end{array} \right.
 \end{array}$$

$$\begin{array}{r}
 21 \\
 342 \\
 3 - \frac{84}{4} - \frac{20}{10} \\
 \hline
 5
 \end{array}$$

$$\frac{306}{10}$$

$$\begin{array}{r}
 84 \\
 14 \\
 \hline
 336 \\
 -30 \\
 \hline
 306 \\
 84 \\
 15 \\
 \hline
 69
 \end{array}$$

$$\begin{array}{r}
 -\frac{306}{40}s^2 + \frac{69}{5} \\
 -\frac{306}{40}s^2 + \frac{306}{40} \\
 \hline
 \frac{69}{40}
 \end{array}$$

$$\frac{15}{20} - \frac{84}{20} = -\frac{69}{20}$$

$$y_R(s) = \frac{21}{20} \left\{ \frac{-\frac{9}{10}s^2 - \frac{21}{10}}{s \left[-\frac{21}{10}s^2 - \frac{69}{20} \right]} \right\} = \frac{21}{200} \left\{ \frac{9s^2 + 21}{s \left(\frac{21}{10} \right) \left(s^2 + \frac{69}{21} \right)} \right\} \delta[y_R] = 3$$

$$= \frac{9}{20} \left(\frac{s^2 + 21/9}{s(s^2 + \frac{69}{42})} \right) \text{ this RRL is lossless } \delta[y_R] = \delta[y_i]$$

$$C = 21/40$$

