

Homework 3 due next Monday
continuing lossless synthesis

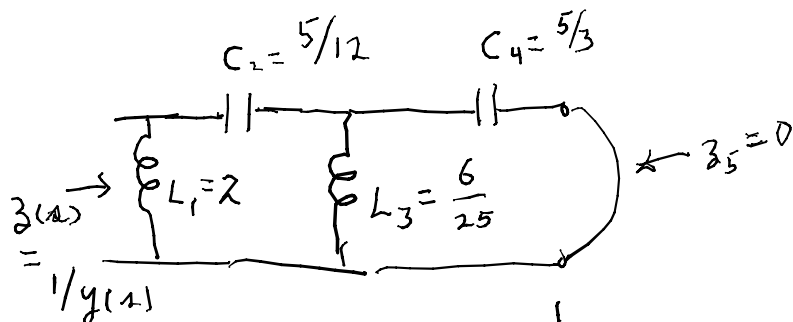
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EE610

$$Z(s) = \frac{3s^3 + 6s}{s^4 + 4s^2 + 3}$$

2nd canon synthesis of this $Z(s) \Rightarrow$ continued fraction expansion about (pole at) 0. $\Rightarrow$ divide lowest powers of s into lowest powers of s ; start with $y(s)$ which has the pole at 0 [note any PR lossless $Z(s)$ has a pole or a zero at $s=0$ as it is an odd function of s , $Z(s) = -Z(-s)$].

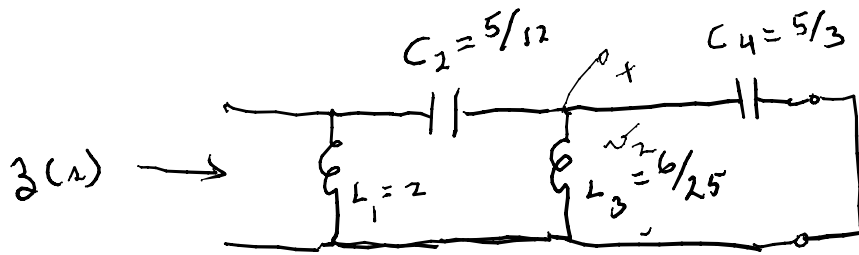
$$\begin{array}{r} 6s + 3s^3 \overline{) 3 + 4s^2 + s^4} \leftarrow y_1 \\ \underline{3 + 3s^2} \\ 5s^2 \\ \underline{5s^2} \\ 0 \end{array} \quad \begin{array}{r} 12 \overline{) 5s} \leftarrow Z_2 \\ \underline{5s} \\ 7s \end{array} \quad \begin{array}{r} 5/2 = \frac{25}{6s} \leftarrow y_3 \\ \underline{(3/5)s} \\ 3s^3 \overline{) 5s^2 + s^4} \\ \underline{5s^2} \\ 0 \end{array} \quad \begin{array}{r} 3 \overline{) 5s} \leftarrow Z_4 \\ \underline{5s} \\ 0 \end{array}$$

0 + 0 remainder
35



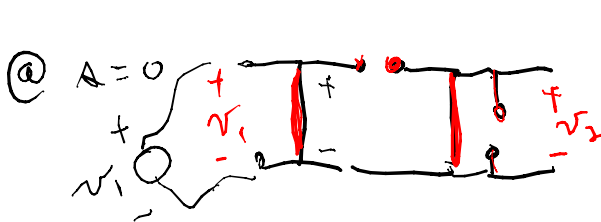
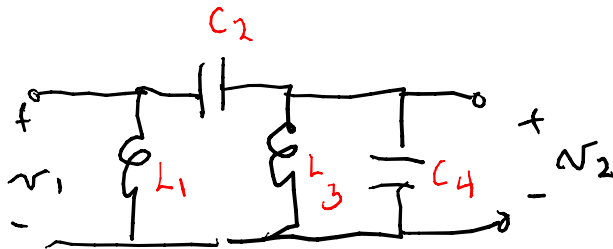
$$\begin{aligned} Z(s) &= \frac{1}{y(s)} \\ &= \frac{3s^3 + 6s}{s^4 + 4s^2 + 3} = \frac{1}{\frac{1}{2s} + \frac{1}{\frac{12}{5s} + \frac{1}{\frac{25}{6s} + \frac{1}{\frac{3}{5s} + 0}}}} \end{aligned}$$

continued fraction expansion about $s=0 \Rightarrow$
2nd canon circuit; zeros of $\frac{V_{out}}{V_{in}}$ all at $s=0$



uses only 4
reactive elements
(L's & C's)

(use for high-pass filters
as series arms open & shunt
arms short @ $\omega = 0$)

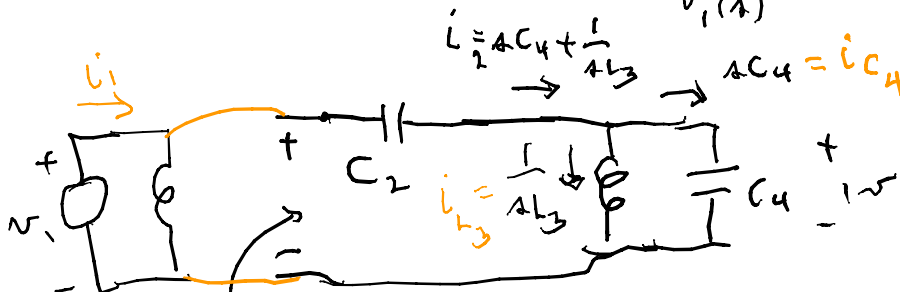


has 3 zeros of transmission
@ $\omega = 0$

$$T(s) = \frac{K_0 s^3}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

to find $T(s) = \frac{v_2(s)}{v_1(s)}$ assume $v_2 = 1V$, find $v_1(s)$ & then

$$T(s) = \frac{1}{v_1(s)} \text{ as linear}$$



$$v_1 = 1 + v_{C2} = 1 + \frac{1}{sC_2} \left(sC_4 + \frac{1}{sL_3} \right) = \frac{s^2 \left(1 + \frac{C_4}{C_2} \right) L_2 L_3 + 1}{s^2 C_2 L_3}$$

$$\frac{v_2}{v_1} = T(s) = \frac{s^2 C_2 L_3}{[s^2 (1 + \frac{C_4}{C_2}) C_2 L_3 + 1]}$$

Richards' function p. 361 given $z(s)$

$$R(s) = \frac{k z(s) - s z(k)}{k z(k) - s z(s)}$$

if $z(s)$ is PR & k is real & positive then $R(s)$ is PR
 a pole at $s = k$ cancels with a zero at $s = k$
 or degree looks like it stays the same.

If $s = k$ is also a zero of $Ev z(s)$ then the degree goes down by 1 as another pole and zero cancel.

Ex: $z(s) = \frac{2s}{3s^2 + 1}$; $Ev z(s) \equiv 0$, so we can choose any $k > 0$

choose $k = 4$; $z(4) = \frac{8}{49}$

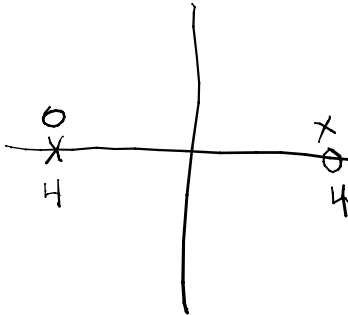
$$R(s) = \frac{4 \left(\frac{2s}{3s^2 + 1} \right) - s \left(\frac{8}{49} \right)}{4 \times \frac{8}{49} - \frac{2s^2}{3s^2 + 1}} = \frac{8s - \frac{24}{49}s - \frac{8}{49}s}{\frac{32}{49} \times 3s^2 + \frac{32}{49} - 2s^2}$$

$$= \frac{s \left(8 - \frac{8}{49} - \frac{24}{49}s^2 \right)}{\frac{32}{49} + \frac{96}{49}s^2 - 2s^2} = \frac{s (8 \times 49 - 8 - 24s^2)}{32 + [96 - 98]s^2}$$

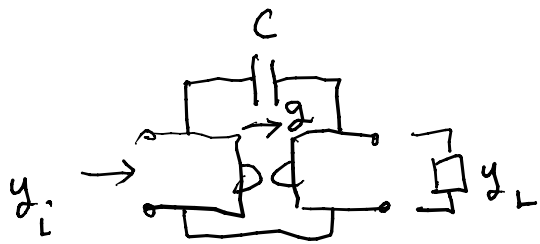
$$\begin{array}{r} 16 \\ 3 \\ \hline 8 \\ 49 \\ 12 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 49 \\ 78 \\ \hline 392 \end{array}$$

$$= \frac{a \left[\frac{8+48}{24} - a^2 \right] 24}{\frac{32}{2} - a^2} = 24a \quad \text{in PR}$$



a nice circuit for this is



$$Y = \begin{bmatrix} sC & g - sC \\ -g - sC & sC \end{bmatrix}$$

$$i_i = \frac{y_{11} y_L + \Delta Y}{y_{22} + y_L}$$

solve for \$y_L\$

$$y_L y_i + y_{22} y_i = y_{11} y_L + \Delta Y$$

$$(y_i - y_{11}) y_L = \Delta Y - y_{22} y_i$$

$$Y = \begin{bmatrix} 0 & g \\ -g & 0 \end{bmatrix} + sC \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 = -y_L v_2 \end{bmatrix} = Y \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$(-y_{22} - y_L) v_2 = y_{21} v_1$$

$$i_1 = \left[y_{11} + y_{12} \left(\frac{-y_{21}}{y_{22} + y_L} \right) \right] v_1$$

$$= \frac{\Delta Y + y_{11} y_L}{y_{22} + y_L} v_1$$

$$y_L = \frac{\Delta y - y_{22} y_i}{y_i - y_{11}},$$

$$= \frac{g^2 - ac y_i(s)}{y_i(s) - ac}$$

$$\begin{aligned} \Delta y &= a^2 c^2 - (g-ac)(-g-ac) \\ &= a^2 c^2 + g^2 - (-ac)^2 \\ &= g^2 \end{aligned}$$

to identify with

$$R(s) = \frac{bz(s) - az(s)}{bz(s) - az(s)}$$