

10/09/96

EE 610

For homework assume $C_{gA} = C_{gD} = C_g$

$$v_2 = 2v_3 - v_1, \quad \frac{v_2}{v_1} = T(s) = \frac{P(-s)}{P(s)} \quad \text{for all-pass } P(s) \text{ Hurwitz polynomial}$$

$$\Rightarrow T(s)v_1 = 2v_3 - v_1$$

$$\text{solve for } \frac{v_3}{v_1} \Rightarrow T(s) = 2\frac{v_3}{v_1} - 1 \Rightarrow \frac{v_3}{v_1} = \frac{1}{2}(1 + T(s)) = \frac{1}{2} \left(\frac{P(s) + P(-s)}{P(s)} \right)$$

$$\therefore \frac{v_3}{v_1} = \frac{E_v P(s)}{E_v P(s) + \text{od } P(s)} = \frac{E_v P(s)/\text{od } P(s)}{1 + \frac{E_v P(s)}{\text{od } P(s)}} = \frac{R E_v P(s)/\text{od } P(s)}{R + R E_v P(s)/\text{od } P(s)}$$



$$Z(s) = R \frac{E_v P(s)}{\text{od } P(s)} = -Z(-s) \Leftarrow \begin{array}{l} \text{a lossless} \\ \text{impedance} \\ \text{positive-real} \\ Z(s) = -Z(-s) \end{array}$$

It is true that $Z(s) \neq -1$ in $\sigma > 0$ as $P(s)$ is Hurwitz, all poles of $Z(s)$ are on $j\omega$ axis ^{simple} with real residues

$\therefore Z(s)$ is a reactance function $\Rightarrow$ we synthesize by a finite # of L's & C's with positive element values

Synthesis of a reactance function

$Z(s)$; also $y(s) = 1/Z(s)$ is a reactance function

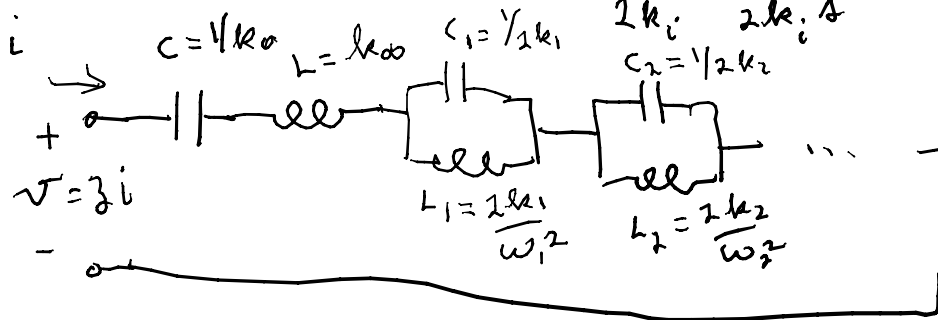
1st Foster synthesis $\Rightarrow$ partial fraction expansion of $Z(s)$

$$Z(s) = \frac{k_0}{s} + k_\infty s + \sum_{i=1}^n \frac{2k_i s}{s^2 + \omega_i^2}$$

$$= \frac{k_0}{s} + k_\infty s + \sum_{i=1}^n \frac{\frac{s}{\frac{1}{2k_i}} + \frac{\omega_i^2}{\frac{1}{2k_i}}}{s^2 + \omega_i^2}$$

as $\frac{k_i}{s-j\omega_i} + \frac{k_i}{s+j\omega_i}$

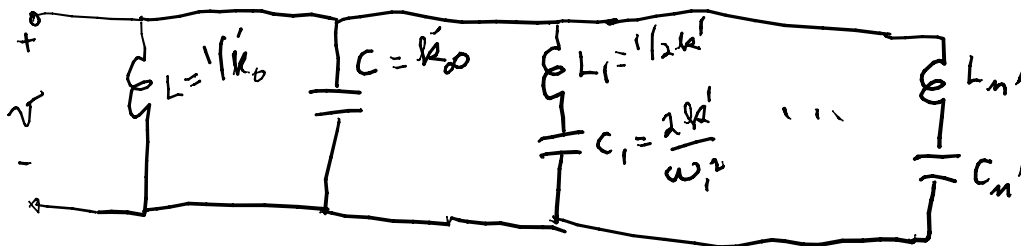
$$= \frac{2k_i s}{(s-j\omega_i)(s+j\omega_i)}$$



2nd Foster $\Rightarrow$ partial fraction expansion of $Y(s) = 1/Z(s)$

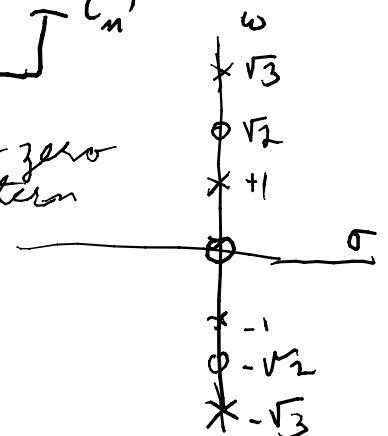
$$Y(s) = \frac{k'_0}{s} + k'_\infty s + \sum_{i=1}^{n'} \frac{2k'_i s}{s^2 + \omega_i'^2} \quad k_i \geq 0$$

$i = YV$



Ex: $Z(s) = \frac{3(s^2 + 2)s}{(s^2 + 1)(s^2 + 3)}$

pole-zero pattern



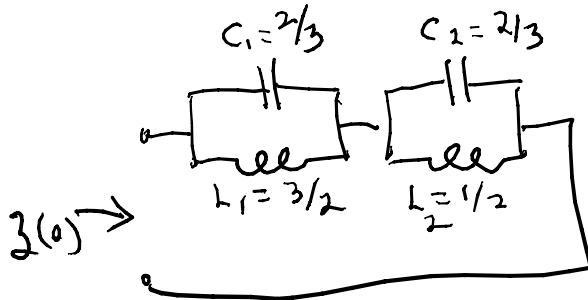
1st Foster

$$Z(s) = \frac{2k_1 s}{s^2 + 1} + \frac{2k_2 s}{s^2 + 3} = \frac{3(s^2 + 2)s}{(s^2 + 1)(s^2 + 3)}$$

$$\times \frac{s^2 + 1}{s} \Big|_{s^2 = -1} = 2k_1 + 2k_2 \left(\frac{s^2 + 1}{s^2 + 3} \right) \Big|_{s^2 = -1} = \frac{3(s^2 + 2)}{s^2 + 3} \Big|_{s^2 = -1} = \frac{3(-1 + 2)}{(-1 + 3)} = 3/2$$

$$\times \frac{s^2 + 3}{s} \Big|_{s^2 = -3} + 2k_2 = \frac{3(s^2 + 2)}{s^2 + 1} \Big|_{s^2 = -3} = \frac{3(-3 + 2)}{(-3 + 1)} = 3/2$$

$$Z(s) = \frac{\frac{3}{2}s}{s^2 + 1} + \frac{\frac{3}{2}s}{s^2 + 3} = \frac{\frac{3}{2}s(s^2 + 3) + \frac{3}{2}s(s^2 + 1)}{(s^2 + 1)(s^2 + 3)} = \frac{3s^3 + 6s}{(s^2 + 1)(s^2 + 3)}$$

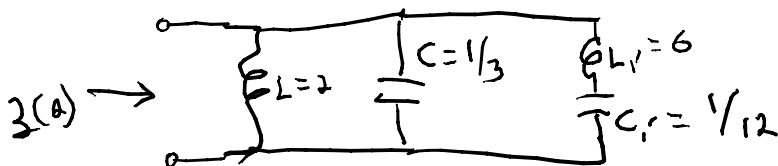


1st Foster

2nd Foster: $Y(s) = \frac{1}{Z(s)} = \frac{(s^2 + 1)(s^2 + 3)}{3s(s^2 + 2)}$

$$Y(s) = \frac{1/2}{s} + \frac{s}{3} + \frac{2k's}{s^2 + 2} ; \quad 2k' = \frac{(s^2 + 1)(s^2 + 3)}{3s^2} \Big|_{s^2 = -2} = \frac{(-1)}{-6} = 1/6$$

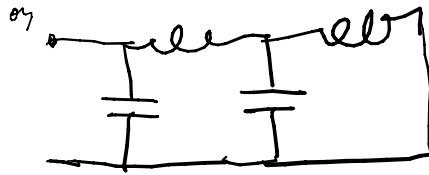
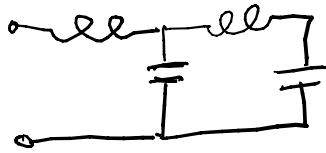
$$= \frac{1}{2s} + \frac{s}{3} + \frac{s/6}{s^2 + 2} = \frac{1}{2s} + \frac{1}{3}s + \frac{1}{6s + \frac{12}{s}} \Rightarrow \text{impedance}$$



2nd Foster
uses only 4 reactive
elements

1st case $\rightarrow$ continued fraction expansion about $s = \infty$

$$Z(s) = k_{\infty} s + \dots$$



$$Z(s) = \frac{3s(s^2+2)}{(s^2+1)(s^2+3)}$$

$$3s(s^2+2)$$

$$Z(s) = \frac{3s^3+6s}{s^4+4s^2+3} \quad \text{no pole at } \infty$$

$$y(s) = \frac{s^4+4s^2+3}{3s^3+6s} = \frac{1}{3}s + y_1$$

$$y_1 = \frac{s^4+4s^2+3}{3s^3+6s} - \frac{1}{3}s = \frac{s^4 - s^4 + 4s^2 - 2s^2 + 3}{3s^3+6s} = \frac{2s^2+3}{3s^3+6s}$$

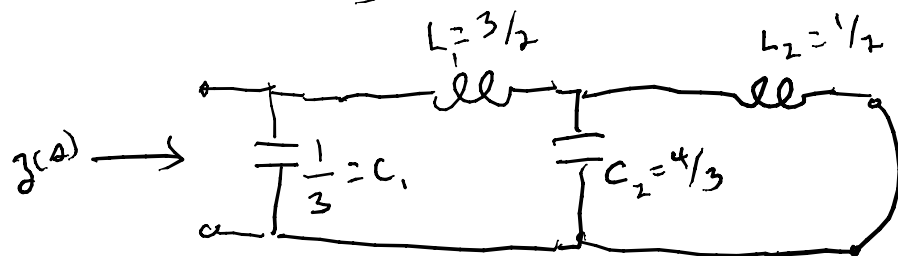
$$z_1(s) = \frac{3s^3+6s}{2s^2+3} = \frac{3}{2}s + z_2$$

(no pole at ∞ , but $=0$ at ∞)

$$\begin{array}{r} \frac{1}{3}s \\ 3s^3+6s \overline{) s^4+4s^2+3} \\ \underline{s^4+2s^2} \\ 2s^2+3 \end{array} \begin{array}{r} (3/2)s \\ 3s^3+6s \overline{) 3s^3+6s} \\ \underline{3s^3+9/2s} \\ 3/2s \end{array} \begin{array}{r} (4/3)s \\ 3/2s \overline{) 2s^2+3} \\ \underline{2s^2} \\ 3 \end{array} \begin{array}{r} \frac{1}{2}s \\ 3 \overline{) (3/2)s} \\ \underline{(3/2)s} \\ 0 \end{array}$$

$$\begin{aligned}
 z(s) &= \frac{1}{\frac{1}{3}s + \frac{1}{(\frac{3}{2})s + \frac{1}{(\frac{4}{3})s + \frac{1}{\frac{1}{2}s + 0}}}} \\
 &\xrightarrow{y_1} \frac{1}{\frac{1}{3}s + \frac{1}{(\frac{3}{2})s + \frac{1}{(\frac{4}{3})s + \frac{1}{\frac{1}{2}s + 0}}}} \\
 &\xrightarrow{z_2} \frac{1}{(\frac{3}{2})s + \frac{1}{(\frac{4}{3})s + \frac{1}{\frac{1}{2}s + 0}}} \\
 &\xrightarrow{y_3} \frac{1}{(\frac{4}{3})s + \frac{1}{\frac{1}{2}s + 0}} \xrightarrow{z_4} \frac{1}{\frac{1}{2}s + 0}
 \end{aligned}$$

continued
fraction
expansion
about $s = \infty$



1st lanes
use only 4
reactive
elements